On the Anisotropy of the Arrival Directions of Galactic Cosmic Rays

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UNIVERSITY OF COPENHAGEN

Galactic Cosmic Rays

- *Standard paradigm:* Galactic CRs accelerated in **supernova remnants**

✔ sufficient power: $\sim 10^{-3} \times M_\odot$ with a rate of ~ 3 SNe per century [Baade & Zwicky'34]

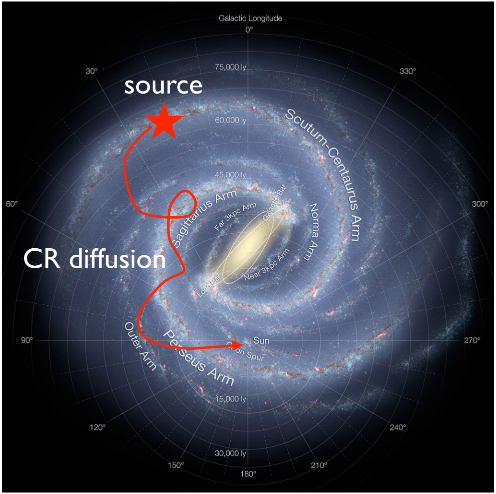
- galactic CRs via diffusive shock acceleration?

$$n_{\rm CR} \propto E^{-\gamma} \qquad \text{(at source)}$$

- energy-dependent **diffusion** through Galaxy

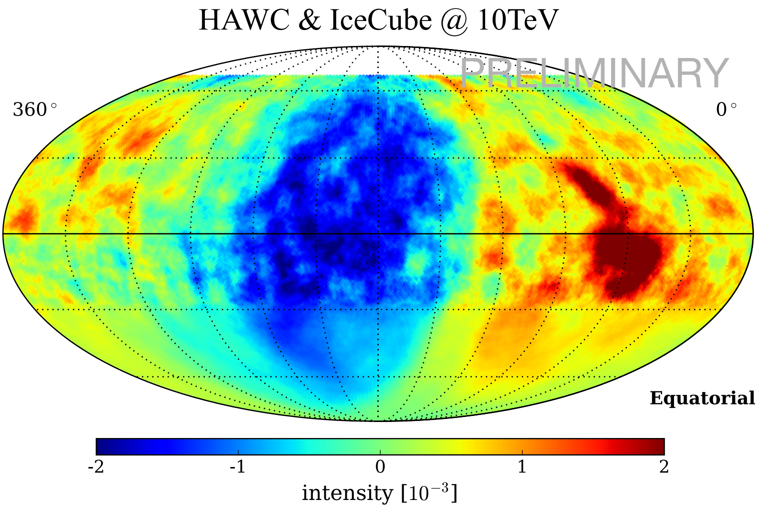
$$n_{\rm CR} \propto E^{-\gamma-\delta} \qquad \text{(observed)}$$

- arrival direction **mostly isotropic**



CR Arrival Directions

Cosmic ray anisotropies up to the level of **one-per-mille** at various energies
(Super-Kamiokande; Milagro; ARGO-YBJ; EAS-TOP, Tibet AS-γ; IceCube; HAWC)



[➜ talk by Dan Fiorino; IceCube & HAWC'17]

Dipole Anisotropy

- spherical harmonic expansion of **relative CR intensity**:

$$I(\alpha,\delta) \simeq 1 + \underbrace{\boldsymbol{\delta}\cdot\mathbf{n}(\alpha,\delta)}_{\text{dipole anisotropy}} + \mathcal{O}\left(\{a_{\ell m}\}_{\ell\geq 2}\right)$$

- expected dipole anisotropy:

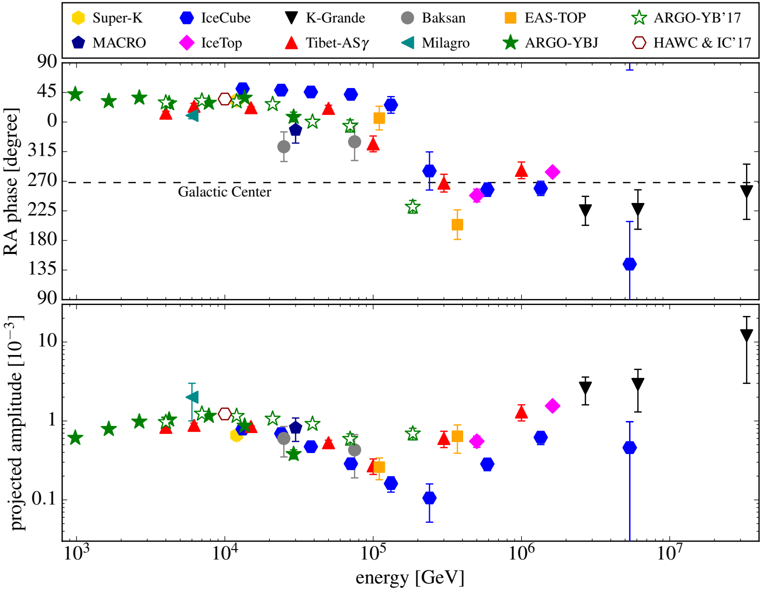
$$\boldsymbol{\delta} = \underbrace{3\mathbf{K}\cdot\nabla\ln n_{\rm CR}}_{\text{CR diffusion}} + \underbrace{(2+\Gamma_{\rm CR})\boldsymbol{\beta}}_{\text{Compton-Getting}}$$

- **Data-driven methods** of anisotropy reconstructions used by ground-based observatories are **only sensitive to dipole along the equatorial plane (EP)** (or, more generally, to all $m \neq 0$ multipoles). [➜ talk by Dan Fiorino]

$$\Delta|\boldsymbol{\delta}_{\rm EP}| \sim \frac{f_{\rm sky}}{\sqrt{N_{\rm tot}}}$$

- **Monte-Carlo-based methods** are sensitive to the full dipole, but are **limited by systematic uncertainties**.

TeV-PeV CR Dipole Anisotropy



Local Magnetic Field

- reconstructed diffuse dipole:

$$\boldsymbol{\delta}^\star = \boldsymbol{\delta} - \underbrace{(2+\Gamma_{\rm CR})\boldsymbol{\beta}}_{\text{Compton-Getting}} = 3\mathbf{K}\cdot\nabla\ln n^\star$$

- projection onto equatorial plane: ➜

$$\boldsymbol{\delta}^\star_{\rm EP} = (\delta^\star_{\rm 0h}, \delta^\star_{\rm 6h})$$

- **strong ordered magnetic fields** in the local environment

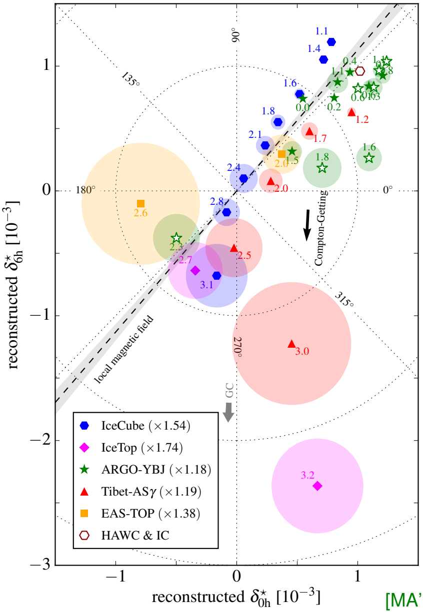
➜ diffusion tensor reduces to **projector**:

[*e.g.* Mertsch & Funk'14; Schwadron *et al.*'14]

$$K_{ij} \to \kappa_\parallel \widehat{B}_i \widehat{B}_j$$

- TeV–PeV dipole data consistent with magnetic field direction inferred by IBEX data [McComas *et al.*'09]

[➜ talk by Eric Zirnstein]



[MA'17]

Known Local Supernova Remnants

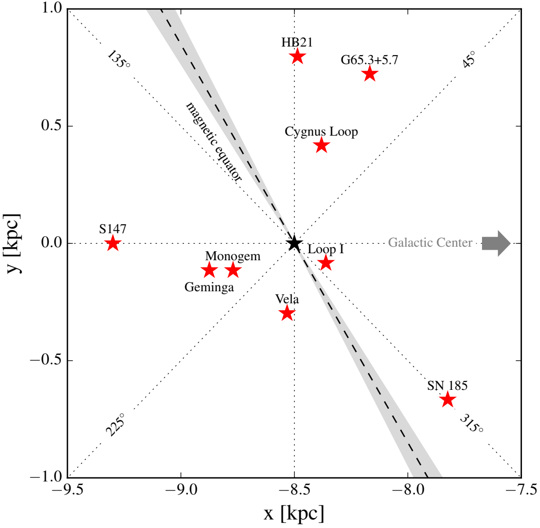
- projection maps source gradient onto $\widehat{\mathbf{B}}$ **or** $-\widehat{\mathbf{B}}$

➜ **dipole phase** α_1 depends on orientation of magnetic hemispheres

- intersection of magnetic equator with Galactic plane defines two source groups:

$$120^\circ \lesssim l \lesssim 300^\circ \rightarrow \alpha_1 \simeq 49^\circ$$

$$-60^\circ \lesssim l \lesssim 120^\circ \rightarrow \alpha_1 \simeq 229^\circ$$



Local Magnetic Field

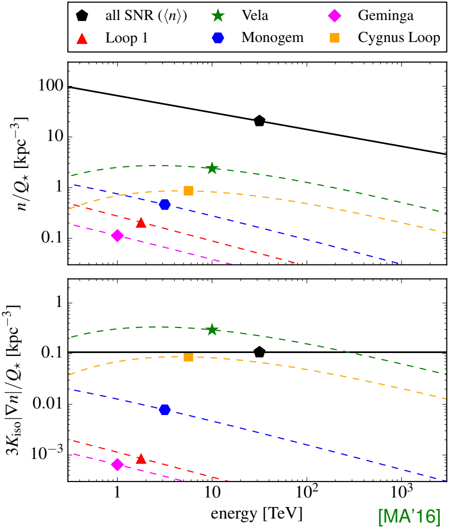
- 1–100 TeV phase indicates dominance of a local source within longitudes:

$$120^\circ \lesssim l \lesssim 300^\circ$$

- **plausible scenario: Vela SNR** [MA'16]
  - *age* : $\simeq 11,000$ yrs
  - *distance* : $\simeq 1,000$ lyrs
  - *SNR rate* : $\mathcal{R}_{\rm SNR} = 1/30\,{\rm yr}^{-1}$
  - *(effective) isotropic diffusion*:

$$K_{\rm iso} \simeq 4 \times 10^{28}(E/3{\rm GeV})^{1/3}{\rm cm}^2/{\rm s}$$

  - *Galactic half height* : $H \simeq 3$ kpc
  - *instantaneous CR emission* ($Q_\star$)



[MA'16]

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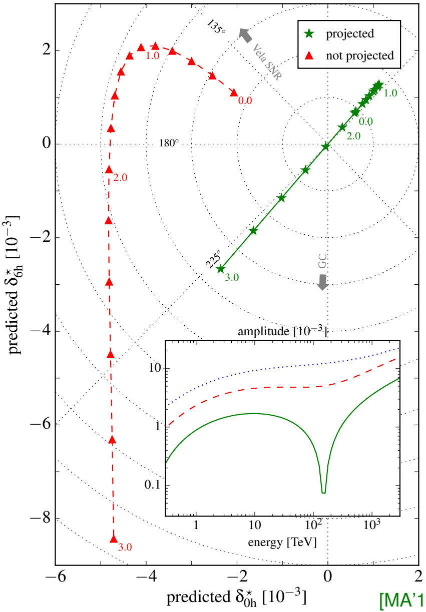
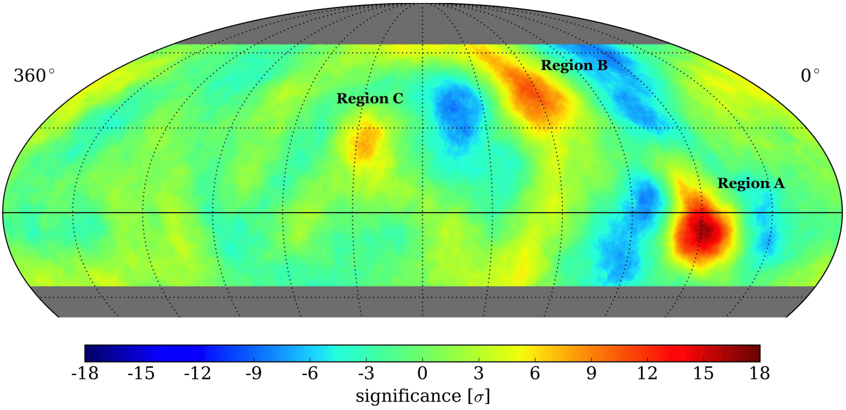


[MA'16]

Small-Scale Anisotropy

Significant TeV small-scale anisotropies down to angular scales of $\mathcal{O}(10)$ degrees.



$E_{\rm CR} \simeq 1$ TeV, $N_{\rm CR} \sim 4.9 \times 10^{10}$ [HAWC'14 (HAWC-111)]

Suggested Origin of Small-Scale Anisotropy

- magnetic reconnections in the heliotail [Lazarian & Desiati'10]
- non-isotropic particle transport in the heliosheath [Desiati & Lazarian'11]
- heliospheric electric field structure [Drury'13]
- non-uniform pitch-angle diffusion [Malkov, Diamond, Drury & Sagdeev'10; Giacinti & Kirk'17] [➜ talk by Gwenael Giacinti]
- non-diffusive CR transport [Salvati & Sacco'08; Drury & Aharonian'08] [Battaner, Castellano & Masip'14; Harding, Fryer & Mendel'16]
- magnetized outflow from old SNRs [Biermann, Becker, Seo & Mandelartz'12] [➜ talk by Julia Tjus]
- strangelet production in molecular clouds or neutron stars [Kotera, Perez-Garcia & Silk '13]

➜ small-scale anisotropies from local magnetic field mapping of a global dipole [Giacinti & Sigl'12; MA'14; MA & Mertsch'15] [Pohl & Rettig'16; López-Barquero, Farber, Xu, Desiati & Lazarian'16]

Angular Power Spectrum

- smooth function $g(\theta, \phi)$ on a sphere can be decomposed in terms of spherical harmonics $Y^\ell_m(\theta, \phi)$:

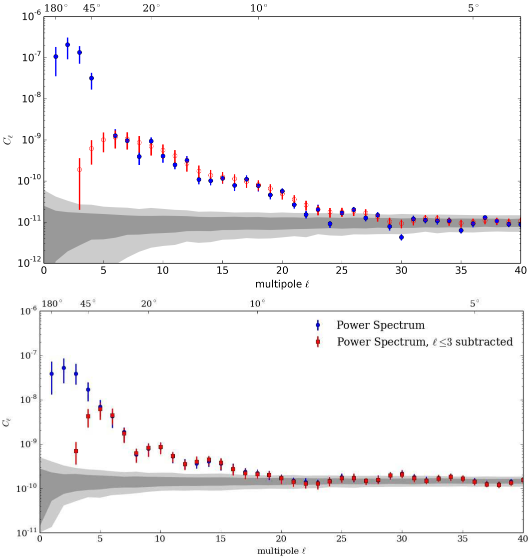
$$g(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_\ell^m(\theta, \phi)$$

- **angular power spectrum**:

$$C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$$

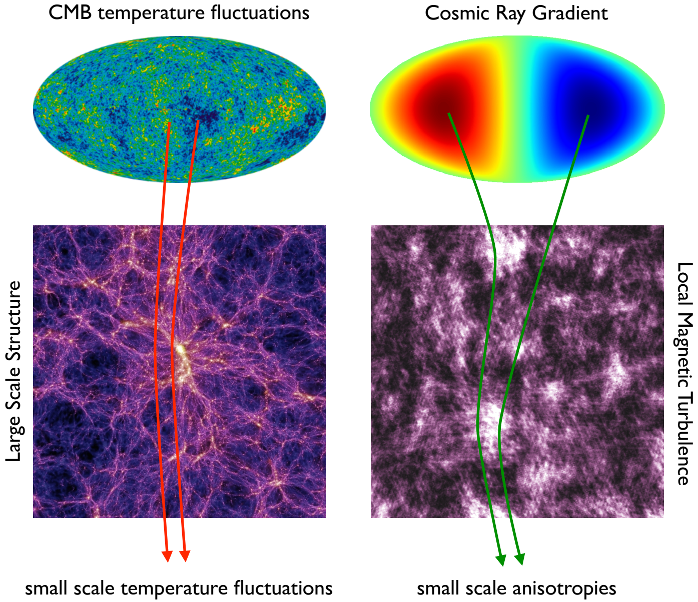
- approximate relation between angular scale and multipole ℓ

$$\Delta\alpha \simeq \frac{180^\circ}{\ell}$$



[IceCube'16 (top) & HAWC'14 (bottom)]

Analogy to Gravitational Lensing



Simulation via CR Backtracking

- (quasi-)stationary solution of the **diffusion approximation**:

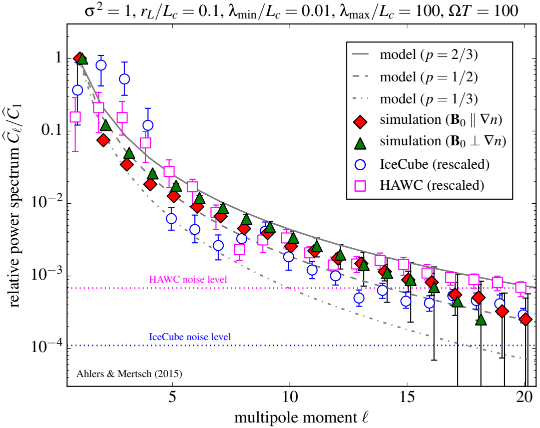
$$4\pi\langle f\rangle \simeq n + \underbrace{\mathbf{r}\nabla n - 3\,\widehat{\mathbf{p}}\,\mathbf{K}\nabla n}_{\text{1st order correction}}$$

- Liouville's theorem:

$$f(t,\mathbf{r}(t),\mathbf{p}(t)) = f(t',\mathbf{r}(t'),\mathbf{p}(t'))$$

- CR backtracking ($T \gg \tau_{\rm diff}$):

$$f(0) \simeq \delta f(-T) + \langle f\rangle(-T)$$



➜ ensemble-averaged power spectrum ($\ell \geq 1$): [MA & Mertsch'15]

$$\frac{\langle C_\ell\rangle}{4\pi} \simeq \int\frac{d\hat{\mathbf{p}}_1}{4\pi}\int\frac{d\hat{\mathbf{p}}_2}{4\pi}P_\ell(\hat{\mathbf{p}}_1\hat{\mathbf{p}}_2)\lim_{T\to\infty}\underbrace{\langle\mathbf{r}_{1i}(-T)\mathbf{r}_{2j}(-T)\rangle}_{\textit{relative}\text{ diffusion}}\frac{\partial_i n\partial_j n}{n^2}$$

Summary

- Observation of CR anisotropies at the level of **one-per-mille** is challenging.
- Reconstruction methods introduce **bias**.
- **Dipole anisotropy** can be understood in the context of standard diffusion theory:
  - TeV-PeV dipole phase aligns with local ordered magnetic field.
  - ➜ **New method** of measuring local magnetic fields
  - Amplitude variations as a result of local sources
  - Plausible & natural candidate: **the Vela supernova remnant**
- Observed CR data shows evidence of **small-scale anisotropy**.
  - Effect of heliosphere? [e.g. review by MA & Mertsch'16]
  - Result of local magnetic turbulence? [Giacinti & Sigl'12; MA'14; MA & Mertsch'15]
  - ✘ Induces cross-talk with dipole anisotropy in limited field of view.

Appendix

Angular Power Spectrum

- Every smooth function $g(\theta, \phi)$ on a sphere can be decomposed in terms of spherical harmonics Y_m^ℓ:

$$g(\theta,\phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_\ell^m(\theta,\phi) \qquad \leftrightarrow \qquad a_{\ell m} = \int \mathrm{d}\Omega (Y_\ell^m)^* (\theta,\phi) g(\theta,\phi)$$

- **angular power spectrum**:

$$C_\ell = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$$

- related to the **two-point auto-correlation function**: ($\mathbf{n}_{1/2}$: unit vectors, $\mathbf{n}_1 \cdot \mathbf{n}_2 = \cos\eta$)

$$\xi(\eta) = \frac{1}{8\pi^2} \int \mathrm{d}\mathbf{n}_1 \int \mathrm{d}\mathbf{n}_2 \delta(\mathbf{n}_1 \mathbf{n}_2 - \cos\eta) g(\mathbf{n}_1) g(\mathbf{n}_2) = \frac{1}{4\pi} \sum_\ell (2\ell+1) C_\ell P_\ell(\cos\eta)$$

→ Note that individual C_ℓ's are **independent** of coordinate system (assuming full sky coverage).

Multipole Cross-Talk

- relative CR intensity (including small-scale structure):

$$I(\alpha,\delta) = 1 + \sum_{\ell\geq 1}\sum_{m\neq 0} a_{\ell m} Y_{\ell m}(\alpha,\pi/2-\delta)$$

- dipole: $a_{1-1} = (\delta_{0\mathrm{h}} + i\delta_{6\mathrm{h}})\sqrt{2\pi/3}$ and $a_{11} = -a^*_{1-1}$

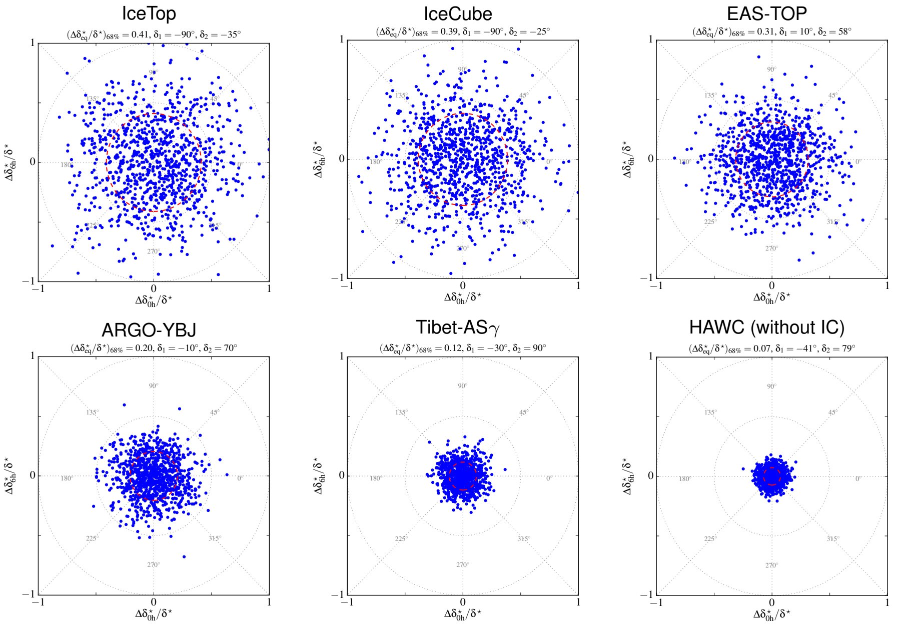
- **traditional dipole analyses** extract amplitude "A_1" and phase "α_1" from data projected into right ascension ($s_{1/2} \equiv \sin\delta_{1/2}$)

$$A_1 e^{i\alpha_1} = \frac{1}{\pi}\int_0^{2\pi}\mathrm{d}\alpha e^{i\alpha}\underbrace{\frac{1}{s_2-s_1}\int_{s_1}^{s_2}\mathrm{d}\sin\delta\, I(\alpha,\delta)}_{\text{projection}}$$

- the presence of high-ℓ multipole moments introduces **cross-talk**

➜ Can now estimate the **systematic uncertainties** of dipole measures from dipole-induced small-scale power spectrum.

Systematic Uncertainty of CR Dipole



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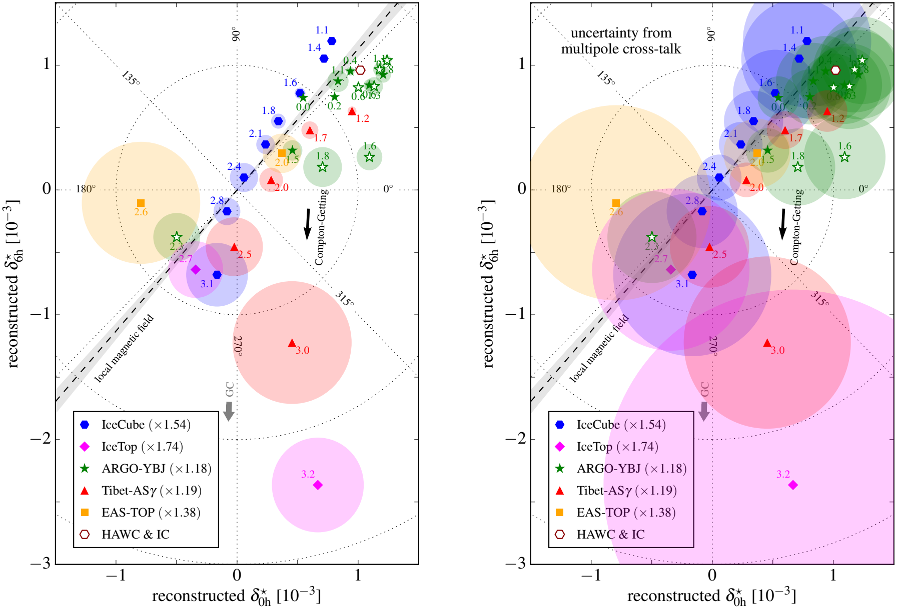


Gedankenexperiment

- **Idea:** local realization of magnetic turbulence introduces small-scale structure

  [Giacinti & Sigl'11]

- Particle transport in (static) magnetic fields is governed by Liouville's equation of the CR's phase-space distribution f:

$$\frac{\mathrm{d}}{\mathrm{d}t} f(t, \mathbf{r}, \mathbf{p}) = 0$$

- "trivial" solution:

$$f(0, \mathbf{0}, \mathbf{p}) = f(-T, \mathbf{r}(-T), \mathbf{p}(-T))$$

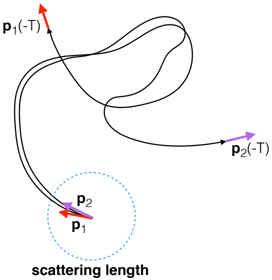
- *Gedankenexperiment*:
  Assume that at look-back time $-T$ initial condition is **homogenous, but not isotropic**:

$$f(0, \mathbf{0}, \mathbf{p}) = \widetilde{f}(\mathbf{p}(-T))$$

Gedankenexperiment

- Initial configuration has power spectrum $\widetilde{C}_\ell$.
- For small correlation angles η flow remains correlated even beyond scattering sphere.
- Correlation function for $\eta = 0$:

$$\xi(0) = \frac{1}{4\pi} \int \mathrm{d}\hat{\mathbf{p}}_1 \widetilde{f}^{\,2}(\mathbf{p}_1(-T))$$



- On **average**, the rotation in an *isotropic* random rotation in the turbulent magnetic field leaves an isotropic distribution on a sphere **invariant**:

$$\langle \xi(0) \rangle = \frac{1}{4\pi} \int \mathrm{d}\hat{\mathbf{p}}_1 \widetilde{f}^{\,2}(\mathbf{p}_1)$$

→ The weighted sum of $\langle C_\ell \rangle$'s remains constant:

$$\frac{1}{4\pi} \sum_{\ell \geq 0} (2\ell + 1) \widetilde{C}_\ell = \frac{1}{4\pi} \sum_{\ell \geq 0} (2\ell + 1) \langle C_\ell(T) \rangle$$

Evolution Model

- Diffusion theory motivates that each $\langle C_\ell \rangle$ decays exponentially with an effective **relaxation rate** [Yosida'49]

$$\nu_\ell \propto \mathbf{L}^2 \propto \ell(\ell+1)$$

- A **linear** $\langle C_\ell \rangle$ evolution equation with generation rates $\nu_{\ell\to\ell'}$ requires:

$$\partial_t \langle C_\ell \rangle = -\nu_\ell \langle C_\ell \rangle + \sum_{\ell' \geq 0} \nu_{\ell' \to \ell} \frac{2\ell'+1}{2\ell+1} \langle C_{\ell'} \rangle \quad \text{with} \quad \nu_\ell = \sum_{\ell' \geq 0} \nu_{\ell \to \ell'}$$

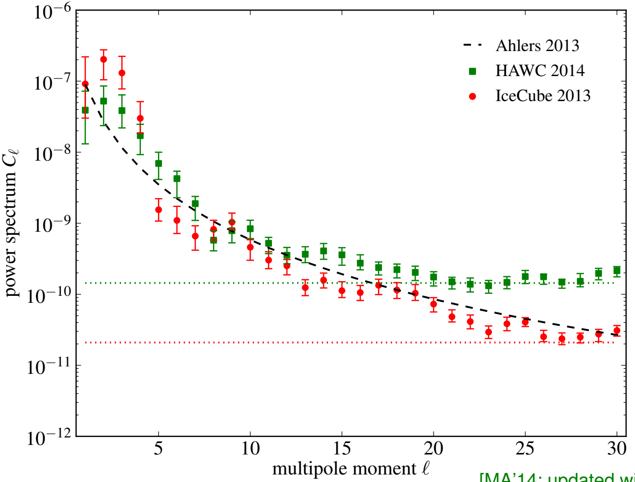
- For $\nu_\ell \simeq \nu_{\ell\to\ell+1}$ and $\widetilde{C}_\ell = 0$ for $l \geq 2$ this has the analytic solution:

$$\langle C_\ell \rangle(T) \simeq \frac{3\widetilde{C}_1}{2\ell+1} \prod_{m=1}^{\ell-1} \nu_m \sum_n \prod_{p=1(\neq n)}^{\ell} \frac{e^{-T\nu_n}}{\nu_p - \nu_n}$$

- For $\nu_\ell \simeq \ell(\ell+1)\nu$ we arrive at a finite asymptotic ratio:

$$\lim_{T\to\infty} \frac{\langle C_\ell \rangle(T)}{\langle C_1 \rangle(T)} \simeq \frac{18}{(2\ell+1)(\ell+2)(\ell+1)}$$

Comparison with CR Data



[MA'14; updated with HAWC data]

$$\lim_{T\to\infty}\frac{\langle C_\ell\rangle(T)}{\langle C_1\rangle(T)} \simeq \frac{18}{(2\ell+1)(\ell+2)(\ell+1)}$$

Local Description: Relative Scattering

- evolution of C_ℓ's: [MA & Mertsch'15]

$$\partial_t \langle C_\ell \rangle = -\frac{1}{2\pi} \int \mathrm{d}\hat{\mathbf{p}}_1 \int \mathrm{d}\hat{\mathbf{p}}_2 P_\ell(\hat{\mathbf{p}}_1 \hat{\mathbf{p}}_2) \langle (\mathbf{p}_1 \nabla f_1 + i\boldsymbol{\omega}\mathbf{L} f_1) f_2 \rangle$$

- large-scale dipole anisotropy gives an effective "source term":

$$-\frac{1}{2\pi} \int \mathrm{d}\hat{\mathbf{p}}_1 \int \mathrm{d}\hat{\mathbf{p}}_2 P_\ell(\hat{\mathbf{p}}_1 \hat{\mathbf{p}}_2) \langle (\mathbf{p}_1 \nabla f_1) f_2 \rangle \to Q_1 \delta_{\ell 1}$$

- BGK-like Ansatz for scattering term ($\langle i\boldsymbol{\omega}\mathbf{L} f \rangle \to -\frac{\nu}{2}\mathbf{L}^2 \langle f \rangle$) [Bhatnagaer, Gross & Krook'54]

$$-\frac{1}{2\pi} \int \mathrm{d}\hat{\mathbf{p}}_1 \int \mathrm{d}\hat{\mathbf{p}}_2 P_\ell(\hat{\mathbf{p}}_1 \hat{\mathbf{p}}_2) \langle (i\boldsymbol{\omega}\mathbf{L} f_1) f_2 \rangle \to \frac{1}{2\pi} \int \mathrm{d}\hat{\mathbf{p}}_1 \int \mathrm{d}\hat{\mathbf{p}}_2 P_\ell(\hat{\mathbf{p}}_1 \hat{\mathbf{p}}_2) \tilde{\nu}(\hat{\mathbf{p}}_1 \hat{\mathbf{p}}_2) \mathbf{L}^2 \langle f_1 f_2 \rangle$$

- Note that $\tilde{\nu}(1) = 0$ for vanishing regular magnetic field.

$$\tilde{\nu}(x) \simeq \nu_0 (1 - x)^p$$

Cosmic Ray Dipole Anisotropy

- cosmic-ray (CR) arrival directions described by **phase-space distribution**

$$f(t,\mathbf{r},\mathbf{p}) = \underbrace{\phi(t,\mathbf{r},p)/(4\pi)}_{\text{monopole}} + 3\,\widehat{\mathbf{p}}\,\underbrace{\boldsymbol{\Phi}(t,\mathbf{r},p)/(4\pi)}_{\text{dipole}} + \ldots$$

- local CR spectral density $[\mathrm{GeV}^{-1}\mathrm{cm}^{-3}]$

$$n(p) = p^2\underbrace{\phi(t,\mathbf{r}_\oplus,p)}_{\propto p^{-(\Gamma_{\rm CR}+2)}} \propto p^{-\Gamma_{\rm CR}}$$

- in the absence of sources, follows Liouville's equation ($\dot{f} = 0$)

➜ quasi-stationary dipole ($\partial_t\boldsymbol{\Phi} \simeq 0$):

$$\underbrace{\partial_t\phi \simeq \nabla_{\mathbf{r}}(\mathbf{K}\nabla_{\mathbf{r}}\phi)}_{\text{diffusion equation}} \quad \text{and} \quad \underbrace{\boldsymbol{\Phi} \simeq -\mathbf{K}\nabla_{\mathbf{r}}\phi}_{\text{Fick's law}}$$

- diffusion tensor $\mathbf{K}$:

$$K_{ij} = \kappa_\parallel \widehat{B}_i\widehat{B}_j + \kappa_\perp(\delta_{ij} - \widehat{B}_i\widehat{B}_j) + \kappa_A\epsilon_{ijk}\widehat{B}_k$$

➜ **dipole anisotropy:** $\delta = 3\mathbf{K}\cdot\nabla_{\mathbf{r}}\ln n$

Compton-Getting Effect

- phase-space distribution is **Lorentz-invariant**

$$f^\star(\mathbf{p}^\star) = f(\mathbf{p})$$

- consider **relative motion of observer** ($\boldsymbol{\beta} = \mathbf{v}/c$) in plasma rest frame ($\star$):

$$\mathbf{p}^\star = \mathbf{p} + p\boldsymbol{\beta} + \mathcal{O}(\beta^2)$$

- Taylor expansion:

$$f(\mathbf{p}) \simeq f^\star(\mathbf{p}) + (\mathbf{p}^\star - \mathbf{p})\nabla_{\mathbf{p}^\star} f^\star(\mathbf{p}) + \mathcal{O}(\beta^2) \simeq f^\star(\mathbf{p}) + p\boldsymbol{\beta}\nabla_{\mathbf{p}^\star} f^\star(\mathbf{p}) + \mathcal{O}(\beta^2)$$

➜ splitting in ϕ and $\boldsymbol{\Phi}$ is **not invariant**:

$$\phi = \phi^\star \qquad \text{and} \qquad \Phi = \Phi^\star + \frac{1}{3}\boldsymbol{\beta}\frac{\partial\phi^\star}{\partial \ln p}$$

- remember: $\phi \sim p^{-2} n_{\rm CR} \propto p^{-2-\Gamma_{\rm CR}}$

$$\boldsymbol{\delta} = \boldsymbol{\delta}^\star + \underbrace{(2+\Gamma_{\rm CR})\boldsymbol{\beta}}_{\text{Compton-Getting effect}}$$