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CONSTRAINING THE FLAVOR STRUCTURE OF  
LORENTZ VIOLATION HAMILTONIAN WITH  
THE MEASUREMENT OF ASTROPHYSICAL  
NEUTRINO FLAVOR COMPOSITIONS

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arXiv: 1704.04027

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# OUTLINE

- Introduction
  - Lorentz violation effects to neutrino flavor transitions
  - Lorentz violation and the current IceCube results on astrophysical neutrino flavor compositions
  - IceCube-Gen2 and its potential of constraining Lorentz Violation Hamiltonian
  - Summary
-

# INTRODUCTION

- Flavor discrimination

Probe the flavor transition,  
ex:  $\nu$  oscillation,  $\nu$  decay,  
Lorentz Violation(LV)

$\Phi_e^0 : \Phi_\mu^0 : \Phi_\tau^0$

Generate the source



$\Phi_e : \Phi_\mu : \Phi_\tau$

detected at the  
Earth

$$(\Phi_e : \Phi_\mu : \Phi_\tau) = P_{\alpha\beta} (\Phi_e^0 : \Phi_\mu^0 : \Phi_\tau^0)$$

# INTRODUCTION

- There are two types of astrophysical sources in proton-proton collision and proton- $\gamma$  collision:

- $PP \rightarrow (\pi^+, \pi^-, \pi^0) + X$

$$\begin{aligned} \pi^+ &\rightarrow \mu^+ + \nu_\mu \\ \mu^+ &\rightarrow \bar{\nu}_\mu + e^+ + \nu_e \end{aligned}$$

$$\begin{aligned} \pi^- &\rightarrow \mu^- + \bar{\nu}_\mu \\ \mu^- &\rightarrow e^- + \bar{\nu}_e + \nu_\mu \end{aligned}$$

$$(V_e, V_\mu, V_\tau) = (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau) \approx (1, 2, 0)$$

1.  $\pi$  source  $(1/3, 2/3, 0)$
2.  $\mu$  damped source  $(0, 1, 0)$

- $P\gamma \rightarrow \Delta^+ \rightarrow n\pi^+$

$$\begin{aligned} \pi^+ &\rightarrow \mu^+ + \nu_\mu \\ \mu^+ &\rightarrow \bar{\nu}_\mu + e^+ + \nu_e \end{aligned}$$

$$(V_e, V_\mu, V_\tau) \approx (1, 1, 0)$$

$$(\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau) \approx (0, 1, 0)$$

1.  $\pi$  source  $(1/3, 2/3, 0)$
2.  $\mu$  damped source  $(0, 1, 0)$

- We shall focus on pion source from pp collision

$$(V_e, V_\mu, V_\tau) = (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau) \approx (1/3, 2/3, 0)$$

- Defining neutrino flavor fraction:

$$f_\alpha^0 = \Phi^0(\nu_\alpha) / (\Phi^0(\nu_e) + \Phi^0(\nu_\mu) + \Phi^0(\nu_\tau))$$



total flux of neutrinos  
and anti-neutrinos at  
the source

Hence

$$(f_e^0, f_\mu^0, f_\tau^0) = (1/3, 2/3, 0)$$

$$f_\alpha = \Phi(\nu_\alpha) / (\Phi(\nu_e) + \Phi(\nu_\mu) + \Phi(\nu_\tau))$$



total flux of neutrinos  
and anti-neutrinos at  
the terrestrial detector

$$f_\alpha = P_{\alpha\beta} f_\beta^0 \quad P_{\alpha\beta} = P(\nu_\beta \rightarrow \nu_\alpha)$$

with  $(f_e^0, f_\mu^0, f_\tau^0) = (1/3, 2/3, 0)$

$$f_e = 1/3 + (P_{e\mu} - P_{e\tau})/3$$

$$f_\mu = 1/3 + (P_{\mu\mu} - P_{\mu\tau})/3$$

$$f_\tau = 1/3 + (P_{\mu\tau} - P_{\tau\tau})/3$$

A test of  $\mu\tau$  symmetry breaking

# LV EFFECTS TO NEUTRINO FLAVOR TRANSITION

- For neutrinos, the general form of LV Hamiltonian

$$\mathbf{H}_{LV}^\nu = \frac{p_\lambda}{E} \begin{pmatrix} a_{ee}^\lambda & a_{e\mu}^\lambda & a_{e\tau}^\lambda \\ a_{e\mu}^{\lambda*} & a_{\mu\mu}^\lambda & a_{\mu\tau}^\lambda \\ a_{e\tau}^{\lambda*} & a_{\mu\tau}^{\lambda*} & a_{\tau\tau}^\lambda \end{pmatrix} - \frac{p^\rho p^\lambda}{E} \begin{pmatrix} c_{ee}^{\rho\lambda} & c_{e\mu}^{\rho\lambda} & c_{e\tau}^{\rho\lambda} \\ c_{e\mu}^{\rho\lambda*} & c_{\mu\mu}^{\rho\lambda} & c_{\mu\tau}^{\rho\lambda} \\ c_{e\tau}^{\rho\lambda*} & c_{\mu\tau}^{\rho\lambda*} & c_{\tau\tau}^{\rho\lambda} \end{pmatrix}$$

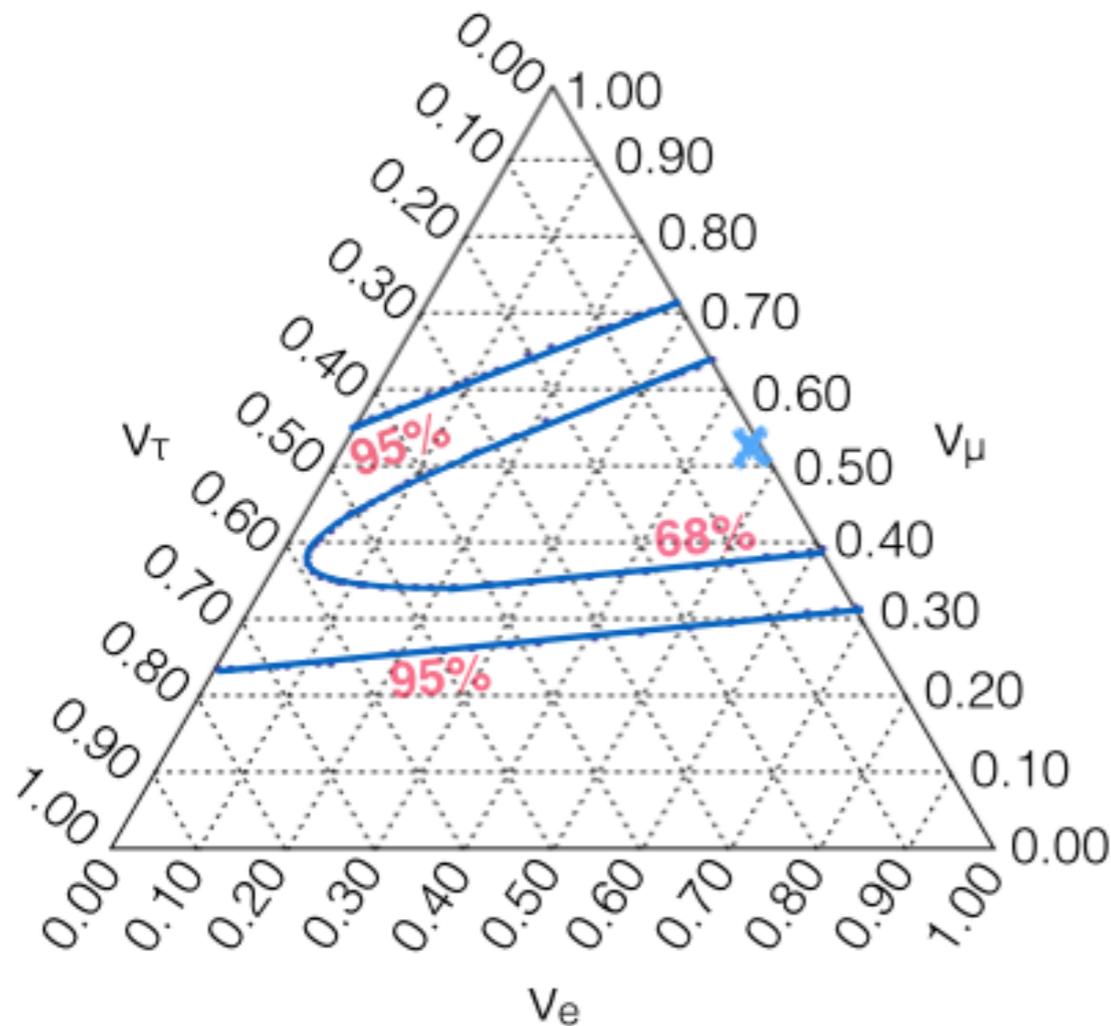
- For rotationally invariant LV effects

$$\mathbf{H}_{LV}^\nu = \begin{pmatrix} a_{ee}^T & a_{e\mu}^T & a_{e\tau}^T \\ a_{e\mu}^{T*} & a_{\mu\mu}^T & a_{\mu\tau}^T \\ a_{e\tau}^{T*} & a_{\mu\tau}^{T*} & a_{\tau\tau}^T \end{pmatrix} - \frac{4E}{3} \begin{pmatrix} c_{ee}^{TT} & c_{e\mu}^{TT} & c_{e\tau}^{TT} \\ c_{e\mu}^{TT*} & c_{\mu\mu}^{TT} & c_{\mu\tau}^{TT} \\ c_{e\tau}^{TT*} & c_{\mu\tau}^{TT*} & c_{\tau\tau}^{TT} \end{pmatrix}$$

$$\mathbf{H}_{LV}^{\bar{\nu}} = - \begin{pmatrix} a_{ee}^T & a_{e\mu}^T & a_{e\tau}^T \\ a_{e\mu}^{T*} & a_{\mu\mu}^T & a_{\mu\tau}^T \\ a_{e\tau}^{T*} & a_{\mu\tau}^{T*} & a_{\tau\tau}^T \end{pmatrix}^* - \frac{4E}{3} \begin{pmatrix} c_{ee}^{TT} & c_{e\mu}^{TT} & c_{e\tau}^{TT} \\ c_{e\mu}^{TT*} & c_{\mu\mu}^{TT} & c_{\mu\tau}^{TT} \\ c_{e\tau}^{TT*} & c_{\mu\tau}^{TT*} & c_{\tau\tau}^{TT} \end{pmatrix}^*$$

Sun-centered celestial equatorial frame  
(T,X,Y,Z) Let us first focus on  $a^T_{\alpha\beta}$

# LORENTZ VIOLATIONS AND CURRENT ICECUBE RESULTS ON ASTROPHYSICAL NEUTRINO FLAVOR COMPOSITION



- $E_\nu$  is between 25 TeV and 2.8 PeV

$$H_{SM} \approx \Delta m_{31}^2 / 2E_\nu$$

Hence  $H_{SM}$  is between

$$5 \times 10^{-26} \text{ GeV}$$

and

$$4.5 \times 10^{-28} \text{ GeV}$$

**Can Lorentz Violation play role in this data?**

# CURRENT BOUNDS ON LORENTZ VIOLATION PARAMETERS

Super-K's result: K.Abe et al. (Super-Kamiokande Collaboration), Phys. Rev. D **91**, 052003(2015)

LV parameter	Limit at 95% C.L.	Best fit	No LV $\Delta\chi^2$	Previous limit	
$e\mu$	$\text{Re}(a^T)$	$1.8 \times 10^{-23}$ GeV	$1.0 \times 10^{-23}$ GeV	1.4	$4.2 \times 10^{-20}$ GeV [1]
	$\text{Im}(a^T)$	$1.8 \times 10^{-23}$ GeV	$4.6 \times 10^{-24}$ GeV		
	$\text{Re}(c^{TT})$	$8.0 \times 10^{-27}$	$1.0 \times 10^{-28}$	0.0	$9.6 \times 10^{-20}$ [1]
	$\text{Im}(c^{TT})$	$8.0 \times 10^{-27}$	$1.0 \times 10^{-28}$		
$e\tau$	$\text{Re}(a^T)$	$4.1 \times 10^{-23}$ GeV	$2.2 \times 10^{-24}$ GeV	0.0	$7.8 \times 10^{-20}$ GeV [2]
	$\text{Im}(a^T)$	$2.8 \times 10^{-23}$ GeV	$1.0 \times 10^{-28}$ GeV		
	$\text{Re}(c^{TT})$	$9.3 \times 10^{-25}$	$1.0 \times 10^{-28}$	0.3	$1.3 \times 10^{-17}$ [2]
	$\text{Im}(c^{TT})$	$1.0 \times 10^{-24}$	$3.5 \times 10^{-25}$		
$\mu\tau$	$\text{Re}(a^T)$	$6.5 \times 10^{-24}$ GeV	$3.2 \times 10^{-24}$ GeV	0.9	...
	$\text{Im}(a^T)$	$5.1 \times 10^{-24}$ GeV	$1.0 \times 10^{-28}$ GeV		
	$\text{Re}(c^{TT})$	$4.4 \times 10^{-27}$	$1.0 \times 10^{-28}$	0.1	...
	$\text{Im}(c^{TT})$	$4.2 \times 10^{-27}$	$7.5 \times 10^{-28}$		

$$H_{\text{SM}} < 5 \times 10^{-26} \text{ GeV}$$

Significant room for  $H_{\text{LV}}$  to play an important role

V. Alan Kostelecký and Neil Russell, Rev. Mod. Phys. **83**, 11(2016)

[1] T. Katori (MiniBooNE Collaboration), Mod. Phys. Lett. A **27**, 1230024(2012)

[2] T. Katori and J. Spitz, in CPT and Lorentz Symmetry VI (World scientific, Singapore, 2014)

# SIMPLE LORENTZ VIOLATION HAMILTONIAN

$$\mathbf{H}_{LV}^\nu = \begin{pmatrix} 0 & 0 & a_{e\tau} \\ 0 & 0 & 0 \\ a_{e\tau}^* & 0 & 0 \end{pmatrix} \Rightarrow P = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{pmatrix}$$

Recall:  $(\Phi_e:\Phi_\mu:\Phi_\tau) = P_{\alpha\beta}(\Phi_e^0:\Phi_\mu^0:\Phi_\tau^0)$

$$(P_{e\mu} - P_{e\tau}) = (P_{\mu\tau} - P_{\tau\tau}) = -1/2$$

$$(P_{\mu\mu} - P_{\mu\tau}) = 1$$

$$(f_e, f_\mu, f_\tau) = (1/6, 2/3, 1/6)$$

Large breaking of  $\mu\tau$  symmetry

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# SIMPLE LORENTZ VIOLATION HAMILTONIAN

$$\mathbf{H}_{LV}^\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & a_{\mu\mu}^T & 0 \\ 0 & 0 & a_{\tau\tau}^T \end{pmatrix} \Rightarrow \mathbf{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, a_{\mu\mu}^T \neq a_{\tau\tau}^T$$

Recall:  $(\Phi_e:\Phi_\mu:\Phi_\tau) = P_{\alpha\beta}(\Phi_e^0:\Phi_\mu^0:\Phi_\tau^0)$

$$(P_{e\mu} - P_{e\tau}) = 0$$

$$(P_{\mu\mu} - P_{\mu\tau}) = 1$$

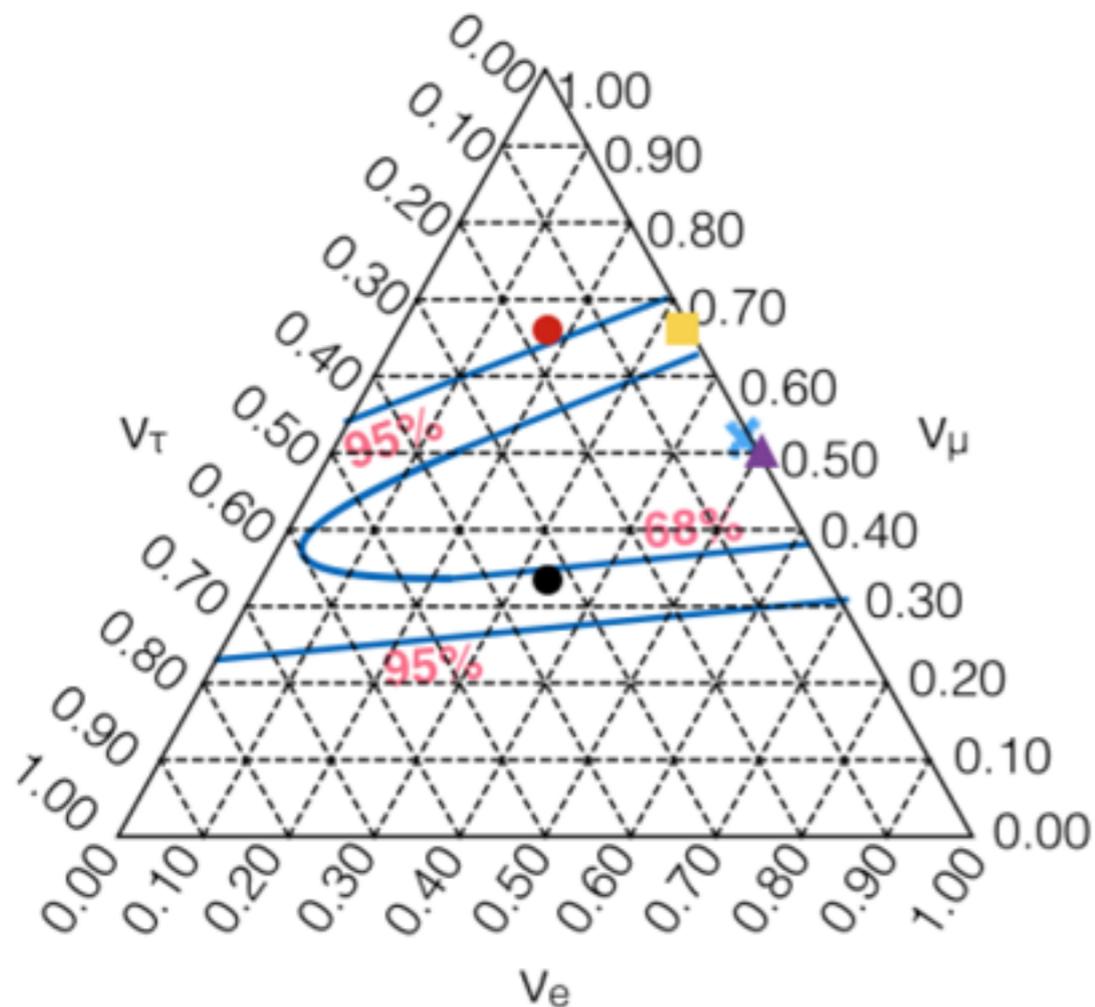
$$(P_{\mu\tau} - P_{\tau\tau}) = -1$$

$$(f_e, f_\mu, f_\tau) = (1/3, 2/3, 0)$$

Large breaking of  $\mu\tau$  symmetry

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# COMPARISONS OF SPECIAL CASES WITH RECENT ICECUBE MEASUREMENT OF ASTROPHYSICAL NEUTRINO FLAVOR COMPOSITION



RED:  $a_{e\tau}^T, a_{e\tau}^{T*} \neq 0$

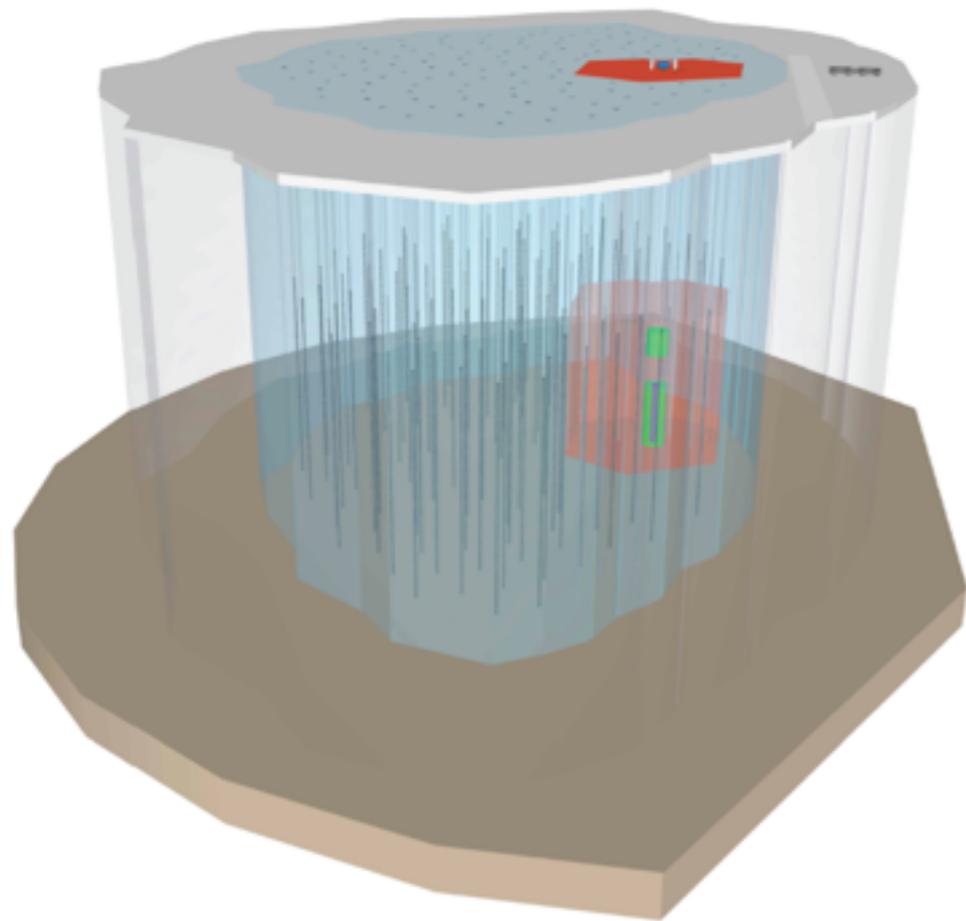
Yellow:  $a_{\mu\mu}^T, a_{\tau\tau}^T \neq 0$

Purple:  $a_{e\mu}^T, a_{e\mu}^{T*} \neq 0$

Black:  $a_{\mu\tau}^T, a_{\mu\tau}^{T*} \neq 0$

All cases fall into  $2\sigma$  region as other elements grow from zero

# ICECUBE-GEN2 AND ITS POTENTIAL OF CONSTRAINING LORENTZ VIOLATION HAMILTONIAN



IceCube Collaboration (M.G. Aartsen  
(Adelaide U.) et al.), arXiv:1412.5106

~10 km<sup>3</sup> instrumented volume  
~250 m spacing of photo sensors

- A possible IceCube-Gen2 configuration
  - IceCube, in red, and the infill sub-detector DeepCore, in green.
  - blue volume shows the full instrumented next-generation detector, with PINGU displayed in grey as a denser infill extension within DeepCore.
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# SENSITIVITIES OF ICECUBE-GEN2 ON ASTROPHYSICAL NEUTRINO FLAVOR COMPOSITIONS

$$\Phi_\nu(E) = \Phi_0 \left( \frac{100 \text{ TeV}}{E} \right)^\gamma$$

$$\gamma = 2.2 \pm 0.2$$

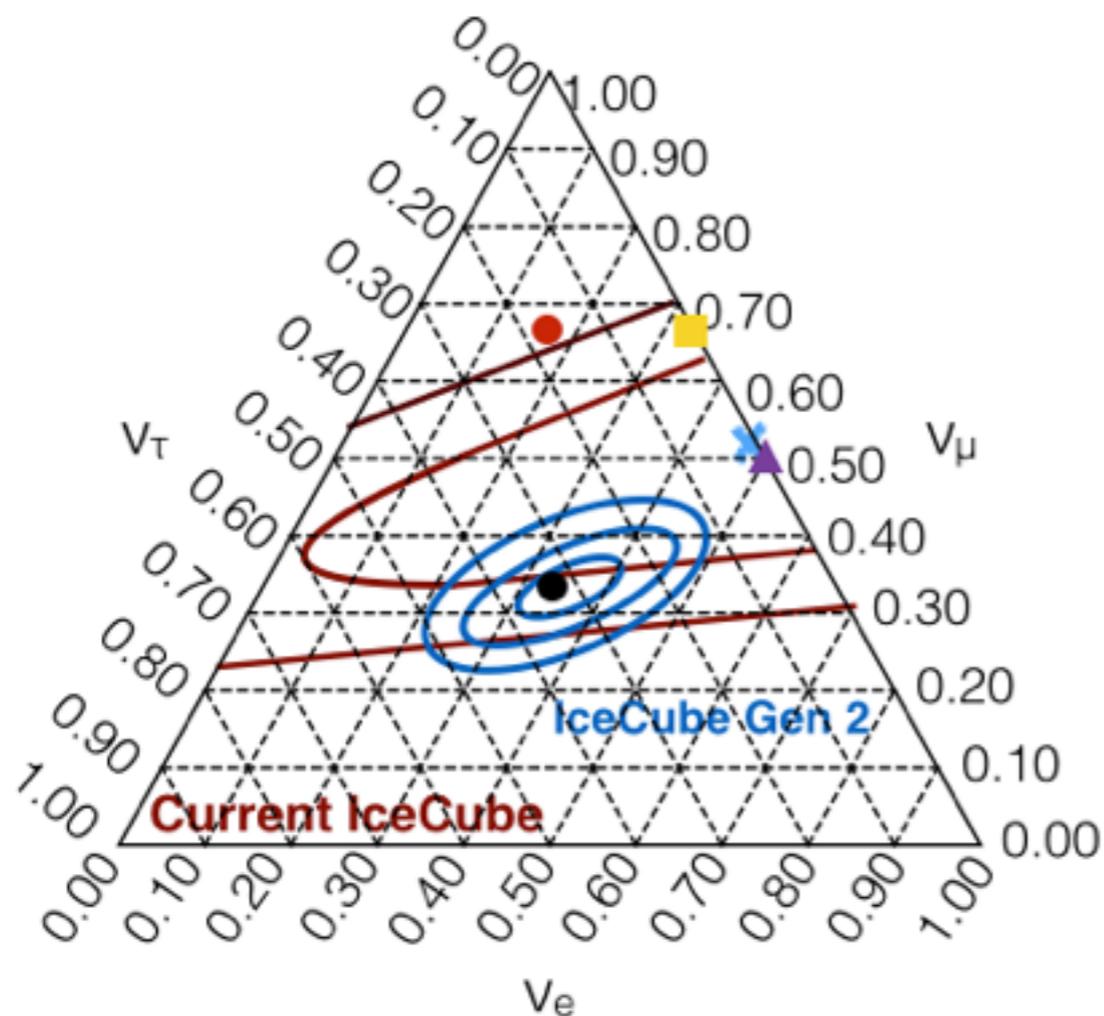
$$\Phi_0 = (5.1 \pm 0.8) \times 10^{-18} \text{ GeV}^{-1} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$$

Pion source from pp collision is assumed

$$E_{\text{th}} = 100 \text{ TeV}$$

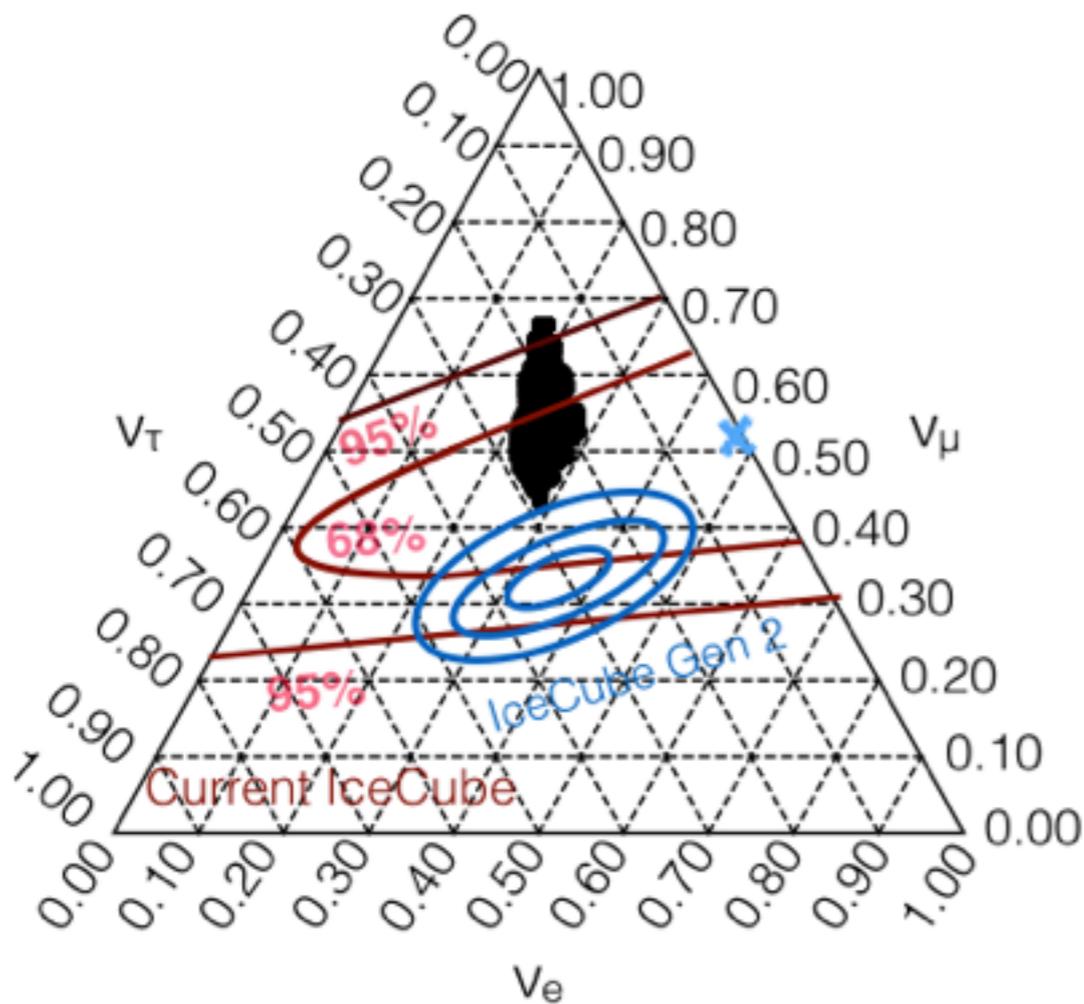
10 years of exposure

$1\sigma$ ,  $2\sigma$ , and  $3\sigma$  regions



I. M. Shoemaker and K. Murase, Phys. Rev. D 93  
085004 (2016) [IceCube-Gen2 regions](#)

SK 95% C.L. limits:  $\text{Re}(a_{e\tau}^T) < 4.1 \times 10^{-23} \text{ GeV}$   $\text{Im}(a_{e\tau}^T) < 2.8 \times 10^{-23} \text{ GeV}$

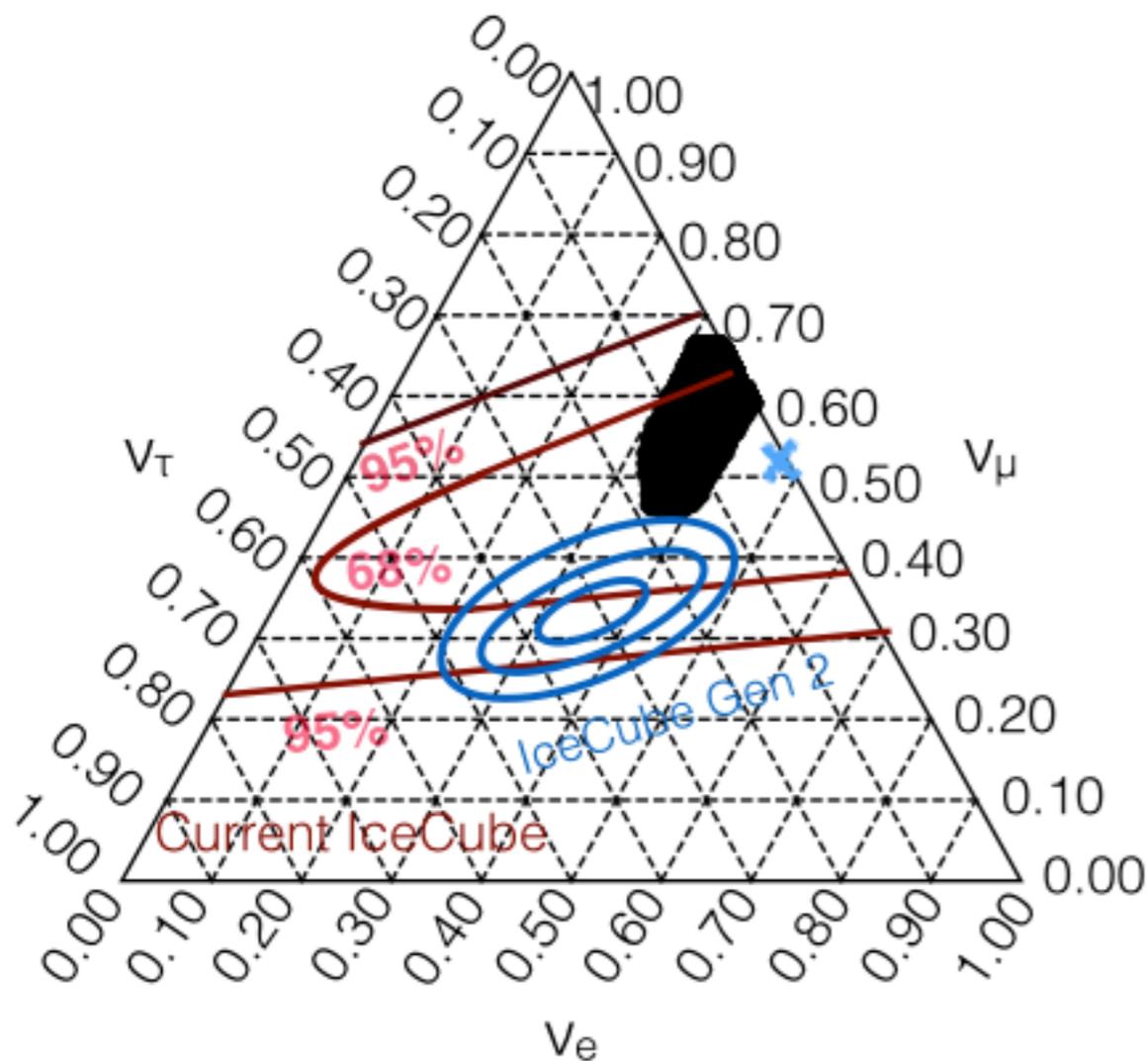


$$\mathbf{H}_{LV}^\nu = \begin{pmatrix} 0 & 0 & a_{e\tau} \\ 0 & 0 & 0 \\ a_{e\tau}^* & 0 & 0 \end{pmatrix}$$

Allow other elements to grow from zero and include the contribution from  $H_{SM}$

The parameter ranges in the table predict the black region of flavor fraction—disfavored at  $3\sigma$ .

$ a_{e\tau}^T $	$5 \times 10^{-24} \text{ GeV}$	$5 \times 10^{-25} \text{ GeV}$
$ a_{e\mu}^T / a_{e\tau}^T $	(0, 0.42)	(0, 0.39)
$ a_{\mu\tau}^T / a_{e\tau}^T $	(0, 0.42)	(0, 0.39)
$ a_{\mu\mu}^T / a_{e\tau}^T $	(0, 0.50)	(0, 0.45)
$ a_{\tau\tau}^T / a_{e\tau}^T $	(0, 0.50)	(0, 0.45)



$$\mathbf{H}_{LV}^\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & a_{\mu\mu}^T & 0 \\ 0 & 0 & a_{\tau\tau}^T \end{pmatrix}$$

Allow other elements to grow from zero and include the contribution from  $H_{SM}$

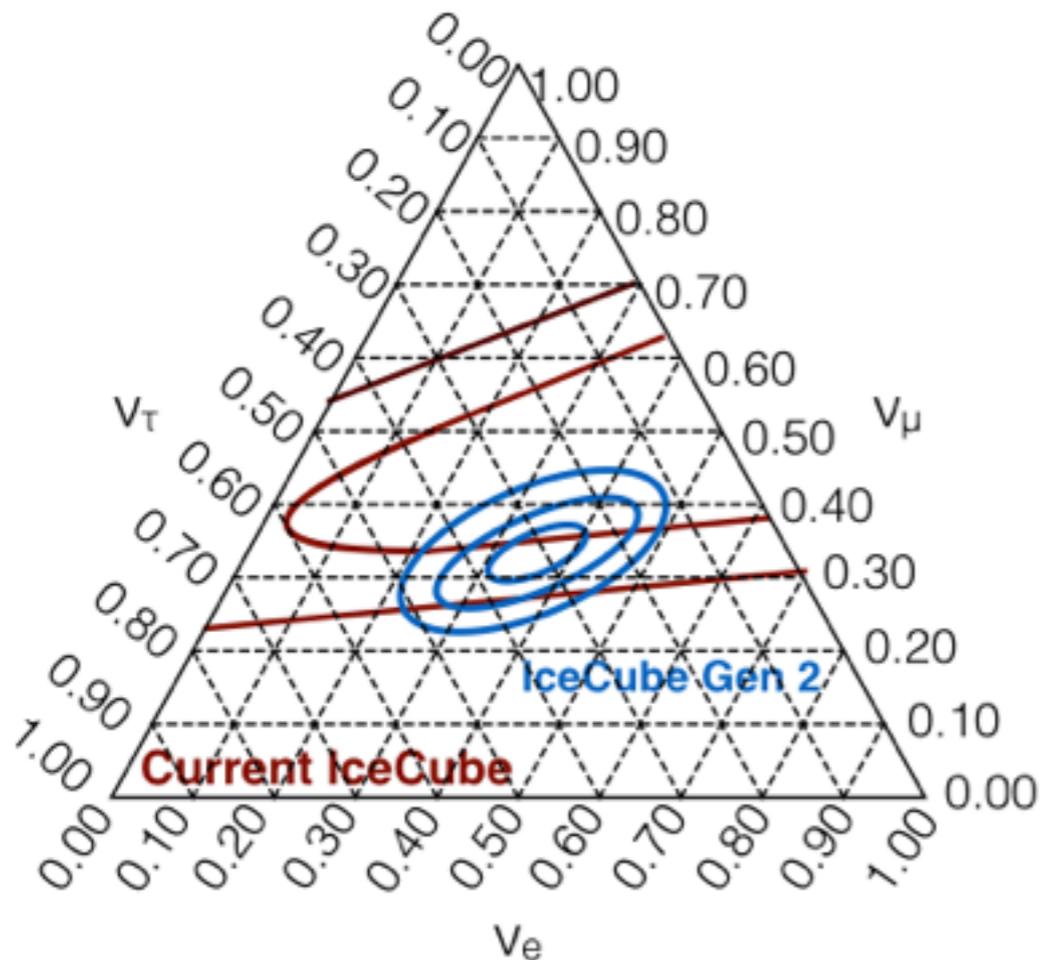
The parameter ranges in the table predict the black region of flavor fraction — disfavored at  $3\sigma$ .

$a_{\mu\mu}^T = 2a_{\tau\tau}^T$	$5 \times 10^{-24}$ GeV	$5 \times 10^{-25}$ GeV
$ a_{e\mu}^T / a_{\mu\mu}^T $	(0, 0.40)	(0, 0.39)
$ a_{e\tau}^T / a_{\mu\mu}^T $	(0, 0.40)	(0, 0.39)
$ a_{\mu\tau}^T / a_{\mu\mu}^T $	(0, 0.40)	(0, 0.39)

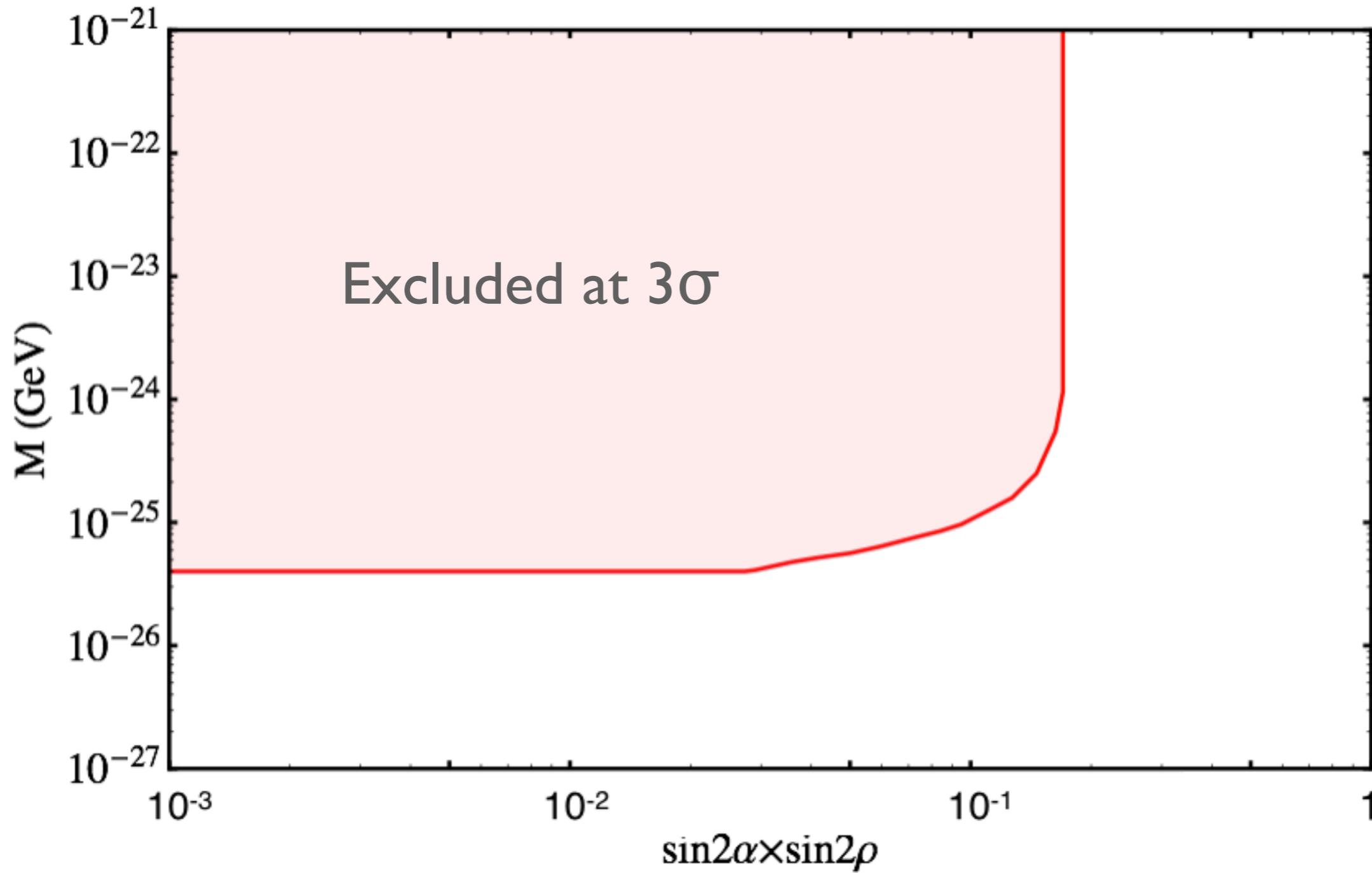
$$H = H_{SM} + H_{LV}$$

$$\mathbf{H}_{LV}^\nu = \begin{pmatrix} 0 & a_{e\mu}^T & a_{e\tau}^T \\ a_{e\mu}^{T*} & a_{\mu\mu}^T & a_{\mu\tau}^T \\ a_{e\tau}^{T*} & a_{\mu\tau}^{T*} & a_{\tau\tau}^T \end{pmatrix}$$

$$= M \begin{pmatrix} 0 & \cos\rho e^{i\sigma} & \sin\rho e^{i\lambda} \\ \cos\rho e^{-i\sigma} & 0 & 0 \\ \sin\rho e^{-i\lambda} & 0 & 0 \end{pmatrix} - M \begin{pmatrix} \gamma & 0 & 0 \\ 0 & \cos 2\alpha & \sin 2\alpha e^{i\beta} \\ 0 & \sin 2\alpha e^{-i\beta} & -\cos 2\alpha \end{pmatrix}$$



Increasing  $M$  until the predicted flavor fraction is out of the IceCube Gen2  $3\sigma$  region.



For most values of  $\sin 2\alpha \times \sin 2\rho$ , the energy scale  $M$  is constrained to be less than few times  $10^{-26}$  GeV

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# Constraint on $C_{\alpha\beta}^{TT}$

$-4EC_{\alpha\beta}^{TT}/3$  replaces  $a_{\alpha\beta}^T$  when the latter is turn off.

if the constraint on  $M$ , which is made of  $a_{\alpha\beta}^T$ , is few times  $10^{-26}$  GeV, the corresponding constraint on  $M'$  (dimensionless quantity made of  $C_{\alpha\beta}^{TT}$ ) is about  $10^{-31}$  with  $E$  chosen as 100 TeV(Threshold energy).

K.Abe et al. (Super-Kamiokande Collaboration), Phys. Rev. D **91**, 052003(2015)

LV parameter	Limit at 95% C.L.	Best fit	No LV $\Delta\chi^2$	Previous limit	
$e\mu$	$\text{Re}(a^T)$	$1.8 \times 10^{-23}$ GeV	$1.0 \times 10^{-23}$ GeV	1.4	$4.2 \times 10^{-20}$ GeV [1]
	$\text{Im}(a^T)$	$1.8 \times 10^{-23}$ GeV	$4.6 \times 10^{-24}$ GeV		
	$\text{Re}(c^{TT})$	$8.0 \times 10^{-27}$	$1.0 \times 10^{-28}$	0.0	$9.6 \times 10^{-20}$ [1]
	$\text{Im}(c^{TT})$	$8.0 \times 10^{-27}$	$1.0 \times 10^{-28}$		
$e\tau$	$\text{Re}(a^T)$	$4.1 \times 10^{-23}$ GeV	$2.2 \times 10^{-24}$ GeV	0.0	$7.8 \times 10^{-20}$ GeV [2]
	$\text{Im}(a^T)$	$2.8 \times 10^{-23}$ GeV	$1.0 \times 10^{-28}$ GeV		
	$\text{Re}(c^{TT})$	$9.3 \times 10^{-25}$	$1.0 \times 10^{-28}$	0.3	$1.3 \times 10^{-17}$ [2]
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$\mu\tau$	$\text{Re}(a^T)$	$6.5 \times 10^{-24}$ GeV	$3.2 \times 10^{-24}$ GeV	0.9	...
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	$\text{Re}(c^{TT})$	$4.4 \times 10^{-27}$	$1.0 \times 10^{-28}$	0.1	...
	$\text{Im}(c^{TT})$	$4.2 \times 10^{-27}$	$7.5 \times 10^{-28}$		

# SUMMARY

- We have introduced Lorentz violation Hamiltonian in neutrino sector.
  - Previous experimental search on Lorentz violation with neutrino is introduced. Previous best limit by Super-Kamiokande experiment is summarized.
  - We have shown that Lorentz violating Hamiltonian with parameters in the above SK limits can change significantly the flavor transition probabilities of high energy astrophysical neutrinos in TeV to PeV energy range.
  - For the pion source induced from pp collisions, Lorentz violating Hamiltonian with large  $\mu\tau$  symmetry breaking effect is more stringently constrained.
  - We show that IceCube-Gen2 can place stringent constraints on the Lorentz violating Hamiltonian.
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