Capture and Decay of Electroweak WIM Ponium

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based on JCAP 1702 (2017) 005 with

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WIMPs

- WIMPs exchange ladder of electroweak gauge bosons

\[ \ldots \]

- in NR limit \((v \sim 10^{-3})\) gives rise to non-local, instantaneous potential

- leads to Sommerfeld enhancement in DM annihilations:

\[ \sigma v = \Gamma |\psi (0)|^2 \]

- WIMP spectrum possesses bound states when WIMP mass sufficiently large relative to mass of electroweak gauge bosons \(\rightarrow\) WIMPorium

\[ \ldots \]

- alternative annihilation channel for DM...significant effect on radiative signals?
**wino**

- SU(2)$_L$ triplet Majorana fermion $\chi_a$, zero hypercharge, mass $M_\chi$
  \[ \mathcal{L} = i\chi_a^\dagger (\bar{\sigma}^\mu \partial_\mu \delta^{ac} + ig\bar{\sigma}^\mu W_\mu^{tb} T_{ac}^b) \chi^c - \frac{1}{2} M_\chi (\chi^a \chi^a + h.c.) \]

- in mass eigenbasis: \( \{\chi^1, \chi^2, \chi^3\} \to \{\chi^0, \chi^\pm\} \)

- mass splitting: \( \delta M \equiv M_{\chi^\pm} - M_{\chi^0} = 165 \text{ MeV} \)

- interactions with electroweak gauge bosons:

- pair states, starting with a pair of neutral winos:
Schrödinger eqtn.

- wino pair states \( \Psi = \begin{pmatrix} \psi_N (\equiv \chi^0 \chi^0) \\ \psi_C (\equiv \chi^+ \chi^-) \end{pmatrix} \)

- in NR limit evolve in the Schrödinger eqtn:
  \[
  i \partial_t \Psi = H^0 \Psi = \left[ -\frac{\nabla^2}{4M_\chi} - \frac{\nabla^2}{M_\chi} + V(r) \right] \Psi
  \]

- under the potential:
  \[
  V_{L+S \text{ even}}(r) = \begin{pmatrix} 0 & -\sqrt{2\alpha_W} e^{-m \frac{r}{r}} \\ -\sqrt{2\alpha_W} e^{-m \frac{r}{r}} & 2\delta M - \frac{\alpha}{r} - \alpha_W c^2_W e^{-m \frac{r}{r}} \end{pmatrix} \quad V_{L+S \text{ odd}}(r) = \begin{pmatrix} 0 & 0 \\ 0 & 2\delta M - \frac{\alpha}{r} - \alpha_W c^2_W e^{-m \frac{r}{r}} \end{pmatrix}
  \]

- initial (neutral) positive-energy scattering state \( \Psi_i \quad E_{CM} = \frac{M_X v^2_{\text{rel}}}{4} \quad V_{L+S \text{ even}} \)
  \[
  \int d^3 r \Psi^\dagger_{i,p} (r) \Psi_{i,p'} (r) = \delta^3 \left( p - p' \right)
  \]
  \[
  H_{L+S \text{ even}}^0 \Psi_i = \frac{M_X v^2_{\text{rel}}}{4} \Psi_i
  \]

  ...radiative transition \( \Delta L = \pm 1 \) to

- purely chargino negative-energy bound state \( \Psi_f \quad E_n \lesssim O \left( \alpha_W^2 M_X \right) \quad V_{L+S \text{ odd}} \)
  \[
  \int d^3 r |\Psi_f (r)|^2 = 1
  \]
  \[
  H_{L+S \text{ odd}}^0 \Psi_f [^{2S+1}L_J] = E_n \Psi_f [^{2S+1}L_J]
  \]
SU(2)\textsubscript{L}-symmetric limit

- high-mass limit in which SU(2)\textsubscript{L} symmetry approximately unbroken:
  \[ M_W, M_Z \ll M_\chi \quad \delta M \to 0 \]
- Coulombic limit:
  \[ V (r) \to -\frac{\alpha_W}{r} \bar{V} \]
- diagonalize the Schrödinger eqtn:
  \[
  -\frac{1}{2\mu} \nabla^2 \Psi - \frac{\alpha_W}{r} \bar{V} \Psi = \begin{cases} 
  \frac{p^2}{2\mu} \Psi \\
  E_n \Psi
  \end{cases}
  \]
  \[ \Psi (r) = \sum_i \eta_i \phi_i (r) \quad \bar{V} \eta_i = \lambda_i \eta_i \]
- Wino:
  \[
  V_{L+S}^{\text{even}} (r) = -\frac{\alpha_W}{r} \begin{pmatrix} 0 & \sqrt{2} \\
  \sqrt{2} & 1 \end{pmatrix}
  \]
  \[ \lambda_1 = 2 \to E_n = -\frac{M_\chi (2\alpha_W)^2}{4n^2} \]
  \[ \lambda_2 = -1 \]
  \[
  V_{L+S}^{\text{odd}} (r) = -\frac{\alpha_W}{r} \begin{pmatrix} 0 & 0 \\
  0 & 1 \end{pmatrix}
  \]
  \[ \lambda = 1 \to E_n = -\frac{M_\chi \alpha_W^2}{4n^2} \]
bound state spectrum

blue, red, green: s, p, d wave. solid, dashed, dotted ranking n from lowest

\[
\begin{align*}
\text{Spin-Singlet Spectrum} & & \text{Spin-Triplet Spectrum} \\
\left| \frac{E_n}{M_{\chi}} \right| = \frac{1}{144} & & 6D \\
\left| \frac{E_n}{M_{\chi}} \right| = \frac{1}{100} & & 5P \\
\left| \frac{E_n}{M_{\chi}} \right| = \frac{1}{64} & & 4P \\
\left| \frac{E_n}{M_{\chi}} \right| = \frac{1}{36} & & 3P 6D \\
\left| \frac{E_n}{M_{\chi}} \right| = \frac{1}{32} & & 5D \\
\left| \frac{E_n}{M_{\chi}} \right| = \frac{1}{16} & & 4S 2P 4D \\
\left| \frac{E_n}{M_{\chi}} \right| = \frac{1}{9} & & 3S 3D \\
\left| \frac{E_n}{M_{\chi}} \right| = \frac{1}{4} & & 2S \\
\left| \frac{E_n}{M_{\chi}} \right| = 1 & & 1S 
\end{align*}
\]

\[E_n = \frac{(2\alpha_W)^2 M_\chi}{4n^2}\] even L+S

\[E_n = \frac{\alpha_W^2 M_\chi}{4n^2}\] odd L+S

high-mass limit
WIM Ponium formation

- initial population of free neutralinos, bound states form via radiative capture

\[
\begin{align*}
\left( \sigma v_{\text{rel}} \right)_{s=1, l=1 \rightarrow 3S_1, n=1} & \propto \frac{\alpha \alpha_W^2}{M_\chi^2 v_{\text{rel}}} e^{-4n\lambda_i/\lambda_f} = \frac{\alpha \alpha_W^2}{M_\chi^2 v_{\text{rel}}} e^{-8} \\
\left( \sigma v_{\text{rel}} \right)_{s=0, l=0 \rightarrow 1P_1, n=2, \sum_m} & \propto \frac{\alpha \alpha_W^2}{M_\chi^2 v_{\text{rel}}} e^{-16}
\end{align*}
\]

- capture into s=1 (spin-triplet) l=0 n=1 bound state (arising from p-wave part of initial state) dominates capture to 2p states — due mostly to suppression $e^{-4n\lambda_i/\lambda_f} = e^{-8n}$

- bound states subsequently decay to lower-energy states or annihilate to SM particles.

- note: detecting photon lines from capture and/or transitions extremely challenging: NFW DM profile $\rho (8.5 \text{kpc}) = 0.4 \text{GeV/cm}^3$

\[
\sigma_{\text{cap}} = 5 \times 10^{-29} \text{cm}^3/\text{s} \quad \rightarrow \quad \mathcal{O} \left( 10^{-3} \right) \text{photons/m}^2/\text{yr}
\]
capture vs. direct annihilation

- leading-order s-wave annihilation into all channels given by diagrams:

\[
\sigma \propto \alpha_W^2 \frac{\alpha W}{M_X^2} \frac{1}{v_{rel}}
\]

\[
\frac{(\sigma v_{rel})_{da}}{(\sigma v_{rel})_{cap}} \propto \frac{\alpha}{\alpha_W} e^{-4\lambda_i/\lambda_f} = \frac{\alpha}{\alpha_W} e^{-8n}
\]

- direct annihilation dominates the radiative capture for the wino, due to factor \(e^{-8n}\)
- in contrast to positronium \(\propto e^{-4n}\)

\textit{dark orange}: tree-level inclusive annihilation \(\rightarrow WW, \gamma Z, \gamma \gamma\).
\textit{blue}: p-wave \(\rightarrow ^3S_1 + \gamma\), (lowest) n=1.
\textit{purple}: d-wave \(\rightarrow ^1P_1 + \gamma\), (n = 2).
\textit{maroon}: s-wave \(\rightarrow ^1P_1 + \gamma\), n = 2.
conclusions

• due to spin statistics, states with odd vs. even $L+S$ experience different effective potentials and form distinct towers of bound states $\rightarrow$ bound spectrum, unsuppressed decay channels different from hydrogen-like atoms.

• wino bound state capture rate subdominant to direct annihilation $\rightarrow$ previous calculations of detectability of e.g. high-energy gamma-ray lines from wino DM should not require significant modification.

• detection of low-energy photon lines from radiative capture and transitions between bound states seem very challenging for wino.

• factors which suppress wino-onium cross section not generic $\rightarrow$ depend sensitively on rep. of DM under the gauge group, and relative masses of DM and force carriers $\rightarrow$ formation of bound states cannot be safely ignored in models with non-trivial dark sectors.

$\rightarrow$See e.g. Cirelli et al JCAP 1705 (2017) 036 : DM charged under dark U(1) $\rightarrow$ formation and decay of DM bound states have significant effect on radiative signals in indirect detection.

$\rightarrow$See e.g. Mitridate et al. 2017: DM fermionic 5plet of SU(2) with zero hypercharge. bound states reduce the DM thermal abundance by about 30%, increasing the DM mass that reproduces the cosmological abundance to about 11.5TeV. significant bound-state corrections to DM indirect detection, characteristic spectrum of mono-chromatic lines around $E \approx (10 \ 80)$ GeV, with rates of experimental interest.
**WIMPonium formation, transitions, annihilation**

- initial population of free neutralinos, bound states form via radiative capture...subsequently decay to lower-energy states or annihilate to SM particles

- continuum-bound and bound-bound transitions in time-ordered perturbation theory.

\[
H = H^0 + V_{\text{rad}}.
\]

\[
V_{\text{rad.}} = \left( - \sum_n \frac{e_n}{M_\chi} A(x_n) \cdot p_n + \sum_n \frac{e_n^2}{2m_n} A(x_n)^2 \right) \mathbb{P}_{\text{CC}}
+ \left( i \sqrt{2} e \alpha_W A(0) \cdot \hat{r} e^{-m_{\text{rr}} r} \right) \mathbb{P}_{\text{NC}}
\]

\[
S_{i, f \gamma} = 2\pi i \delta[M_\chi v^2/4 - E_n - k - P_{\text{BS}}/(4M_\chi)] \left( \sum_n \frac{e_n}{M_\chi} \langle \Psi_f [2S+1 L_J] \gamma(k) | A(x_n) \cdot p_n | \psi_{i,C} \rangle \right)
\]

\[
(d\sigma)_{\nu_{\text{rel}}} / d\Gamma = (2\pi)^2 \mu_f k |M_{i, f \gamma}|^2 d\Omega_k
\]

where \( \mu_f = k E_{\text{BS}}/(k + E_{\text{BS}}) \approx k \)

\[
k = -E_n + M_\chi v_{\text{rel}}^2/4 \quad \text{for capture}
\]

\[
k = E_{n1} - E_{n2} \quad \text{for decay}
\]
annihilation from bound state

- bound states also decay through annihilation to SM final states.

\[ |\psi\rangle = \sqrt{\frac{1}{2\mu}} \int \frac{d^3p}{(2\pi)^3} \psi(p)|p, -p\rangle \quad \text{(distinguishable particles)} \]

\[ \sqrt{\frac{1}{4\mu}} \int \frac{d^3p}{(2\pi)^3} \psi(p)|p, -p\rangle \quad \text{(identical particles)}, \]

\[ \mathcal{M}(B \to f) = \sqrt{\frac{1}{2\mu}} \int \frac{d^3p}{(2\pi)^3} \left[ \frac{1}{\sqrt{2}} \psi_N(p)\mathcal{M}(\chi^0(p)\chi^0(-p) \to f) + \psi_C(p)\mathcal{M}(\chi^+(p)\chi^-(p) \to f) \right] \]

\[ \Gamma = \frac{1}{2M_B} \int d\Pi_n |\mathcal{M}(B \to f)|^2 \]
WIMPorium decays

- decays for lowest-energy bound states:
  - blue $\rightarrow$ 1-3s; yellow $\rightarrow$ 3d; red $\rightarrow$ W+W-

- SU(2)-symmetric limit:
  - $\Gamma_{\text{dec}} \propto \alpha \alpha_W^4 M_\chi$ dominate $L > 0$
  - $\Gamma_{\text{annih}} \propto \alpha_W^{5+2L} M_\chi$

- dominant capture into spin-triplet 1s

- spin-singlet 2p: annihilation decay rate suppressed relative to ED transitions to lower s and d.
detectability

- photons radiated upon capture/transitions could allow study of the QM numbers of DM...constitute a detectable signal? assuming:

  NFW DM profile \( \rho (8.5\text{kpc}) = 0.4\text{GeV/cm}^3 \quad R_s = 20\text{kpc} \)

  \( \sigma_{\text{cap}} = 5 \times 10^{-29}\text{cm}^3/\text{s} \)

  \( \mathcal{O} (10^{-3}) \text{photons/m}^2/\text{yr} \quad \text{at Earth from the Milky Way halo} \)

- from region within 1 degree of Galactic center, rate is instead:

  \( \text{few} \times 10^{-5} \text{photons/m}^2/\text{yr} \)

- rate is prohibitively small for reasonable space-based telescope.

- ground-based gamma-ray telescope with effective areas \( \sim 10^{5-6}\text{m}^2 \)

- however, current and near-future ground based telescopes have low-energy thresholds \( 10 - 20\text{GeV} \)

- need to be lowered by an order of magnitude to observe capture and transition photons from DM \( \mathcal{O} (10) \text{TeV} \rightarrow E_n \sim 1\text{GeV} \quad \text{deepest bound states} \)