Gravitational waves from bubble dynamics: Beyond the Envelope

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Introduction
ERA OF GRAVITATIONAL WAVES

- First detection of GWs from BH binaries → **GW astronomy** has started

- Black hole binary $36M\odot + 29M\odot \rightarrow 62M\odot$

- Frequency $\sim 35$ to $250$ Hz

- Significance $> 5.1\sigma$
ERA OF GRAVITATIONAL WAVES

- Detection of GWs from BH binaries → **GW astronomy** has started

- Next will come **GW cosmology** with space interferometers
  
  e.g. LISA, DECIGO, BBO, ...

- **First-order phase transitions** can be cosmological GW sources

  - Electroweak sym. breaking
    (w/ extensions)
  - B-L breaking
  - PQ sym. breaking
  - GUT breaking
  ... etc.
GRavitational WAves As A Probe to Phase Transition

How (first order) phase transition occurs

- High temperature
  - Trapped at symmetry enhanced point
  - Another extreme appears

- Low temperature
  - false vacuum
  - true vacuum
  - Another extreme becomes stable

Time
GWS AS A PROBE TO PHASE TRANSITION

- Thermal first-order phase transition produces GWs in early universe
  - Field space
  - Position space

false vacuum  true vacuum

released energy

Quantum tunneling

Bubble formation & GW production

true
false  true  (“nucleation”)
GWS AS A PROBE TO PHASE TRANSITION

- Thermal first-order phase transition produces GWs in early universe
  - Field space
    - false vacuum
    - true vacuum
    - released energy
  - Position space
    - GWs $\Box h \sim T$
    - Bubble walls source GWs
    - Bubble formation & GW production

Quantum tunneling
GWs propagates until the present without losing information because of the Planck-suppressed interaction of gravitions.

Phase transition & GW production

propagation

NO scattering = NO information loss

present
GWs can be a unique probe to unknown high-energy particle physics.
GWs behave as non-interacting radiation after production

- Frequency & energy scale correspondence

  Present GW frequency $f_0 \sim 1 \text{ Hz} \times \left( \frac{\beta}{H} \right) \left( \frac{T}{10^7 \text{ GeV}} \right) \Rightarrow \frac{\beta}{H} \sim \mathcal{O}(10^{1-4})$

  Note: Planned space interferometers are sensitive to (sub) Hz GWs.

→ They are sensitive
  to new physics around TeV-PeV range!!
TALK PLAN

1. Introduction

1. Analytic derivation of the GW spectrum

2. Result

3. Conclusion
I. Analytic derivation of GW spectrum
2. ANALYTIC DERIVATION OF THE GW SPECTRUM

- The essence: GW spectrum is determined by

\[ \langle T_{ij}(t_x, x) T_{kl}(t_y, y) \rangle_{\text{ens}} \]

\[ \parallel \text{Ensemble average} \]

[Caprini et al. ‘08]
2. ANALYTIC DERIVATION OF THE GW SPECTRUM

- The essence: GW spectrum is determined by $\langle T_{ij}(t_x, x)T_{kl}(t_y, y) \rangle_{\text{ens}}$

- Why? Note: indices omitted below

Formal solution of EOM: $\Box h \sim T \rightarrow h \sim \int^t dt' \, \text{Green}(t, t')T(t')$

Energy density of GWs ($\sim$ GW spectrum): 

$$\rho_{\text{GW}}(t) \sim \frac{\langle h^2 \rangle_{\text{ens}}}{8\pi G} \sim \int^t dt_x \int^t dt_y \cos(k(t_x - t_y)) \langle TT \rangle_{\text{ens}}$$

same as massless scalar field 

substitute the formal solution

Note: ensemble average because of the stochasticity of the bubbles
2. ANALYTIC DERIVATION OF THE GW SPECTRUM

- The essence: GW spectrum is determined by $\langle T_{ij}(t_x, x)T_{kl}(t_y, y) \rangle_{\text{ens}}$

  → We have to specify energy momentum tensor of the system.
  In thermal phase transition, following 3 ingredients are important.

1. Space-time distribution of the bubbles (nucleation points).
   → Determined by transition rate $\Gamma(t) \sim \Gamma_0 e^{-\beta t}$, $\beta$ : model dependent parameter

2. Energy momentum profile around bubble.
   → We adopt thin wall approximation. Good for low frequencies.

3. Dynamics after bubble collision.
   Envelope approximation is often applied.
   [Kosowsky, Turner, Watkins, PRD45 (’92)]

Beyond envelope!

*Late time GW enhancement was suggested.
*We did a robust estimate of GW spectrum assuming free propagation!
2. ANALYTIC DERIVATION OF THE GW SPECTRUM

- Estimate of the ensemble average $\langle T(t_x, x)T(t_y, y)\rangle_{\text{ens}}$
  
  Note: We have fixed behaviour of energy momentum tensor of the system.

  - Trivial from the definition of ensemble average

\[
\langle T(t_x, x)T(t_y, y)\rangle_{\text{ens}} = \sum \left( \frac{\text{Probability for } T(t_x, x)T(t_y, y) \neq 0}{\text{Value of } T(t_x, x)T(t_y, y) \text{ in that case}} \right) \times \left( \text{Probability that bubble walls are passing through } (t_x, x) \&(t_y, y) \right)
\]
FULL EXPRESSIONS

- Full expression reduces to only $\sim$10-dim. integration [Jinno & Takimoto ‘17]

A part of the result. Calculation is tedious but straightforward.

\[
\Delta^{(d)} = \int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y \\
\int_{0}^{\infty} dr \int_{-\infty}^{t_x} dt_{xn} \int_{-\infty}^{t_y} dt_{yn} \int_{t_{xn}}^{t_x} dt_{xi} \int_{t_{yn}}^{t_y} dt_{yi} \int_{-1}^{1} dc_{xn} \int_{-1}^{1} dc_{yn} \int_{0}^{2\pi} d\phi_{xn,yn} \\
\frac{k^3}{3} \left[ \Theta_{sp}(x_i, y_n)\Theta_{sp}(x_n, y_i) e^{-I(x_i, y_i)} \Gamma(t_{xn})\Gamma(t_{yn}) \right. \\
\times r^2 \left[ j_0(kr) \mathcal{K}_0(n_{xn}, n_{yn}) + \frac{j_1(kr)}{kr} \mathcal{K}_1(n_{xn}, n_{yn}) + \frac{j_2(kr)}{(kr)^2} \mathcal{K}_2(n_{xn}, n_{yn}) \right] \\
\times \partial_{txi} \left[ r_B(t_{xi}, t_{xn})^3 D(t_x, t_{xi}) \right] \partial_{tyi} \left[ r_B(t_{yi}, t_{yn})^3 D(t_y, t_{yi}) \right] \cos(kt_{x,y})
\]
TALK PLAN

1. Introduction
2. Analytic derivation of the GW spectrum
3. Conclusion
NUMERICAL RESULT

- A result

Damping function:
\[ D \sim e^{-t/\tau} \]

Long duration after collision.

Instant disappearance (envelope) \([\text{Jinno & Takimoto '16}]\)
coincide with \([\text{Huber & Konstandin '08}]\)
within factor 2

\([\text{Jinno & Takimoto '17}]\)
NUMERICAL RESULT

- A result

\[ \Omega_{GW} \propto \Delta \]

Linear ($\propto k$) behavior

Sourcing saturates for fixed $k$

Sourcing moves to low $k$

[Jinno & Takimoto '17]
NUMERICAL RESULT

- A result

\[ \Omega_{GW} \propto \Delta \]

Linear (\( \propto k \)) behavior

We confirmed enhancement of GW in low frequencies!!

[Jinno & Takimoto '17]
SUMMARY & FUTURE PROSPECTS

- GW spectrum w/ thin-wall has been derived ANALYTICALLY
  - General nucleation rate & wall velocity & damping of wall energy
- We have obtained GW spectrum in lower frequencies.
  - The huge enhancement compared to the envelope approximation
- Various effects can be implemented
  - Cosmic expansion / Nucl. rate dependence / Wall thickness (w/ truncation?)
- Will deepen our understanding on GW sourcing
Back up
GWS AS A PROBE TO PHASE TRANSITION

★ Rough range of GW amplitude

\[ \Omega_{GM,\text{peak}} \sim O(10^{-2})O(10^{-5})(R_\ast H_\ast)^2 \]

\[ f_{\text{peak}} \sim \frac{1}{H_\ast R_\ast} \frac{T_\ast}{10^8 \text{ GeV}} [\text{Hz}] \]

\[ \Omega_{GW} = \frac{\rho_{GW}}{\rho_{\text{tot}}} \]

\[ H_\ast R_\ast \sim O(10^{-1} \sim 10^{-5}) \]

Model dependent parameter

GW from gravitational waves are sensitive to new physics around TeV-PeV range!!
BEHAVIOR OF SINGLE BUBBLE

- Two main players: scalar field & plasma

- Walls (where the scalar field value changes) want to expand ("pressure")

- Walls are pushed back by plasma ("friction")

- Bubble (wall & surrounding plasma) behavior is determined by

\[
\alpha \equiv \frac{\rho_{\text{released}}}{\rho_{\text{rad}}}
\]

\[
\begin{align*}
\alpha &\gtrsim \mathcal{O}(0.1) : \text{Huge energy release} \\
\alpha &\lesssim \mathcal{O}(0.1) : \text{Small energy release}
\end{align*}
\]
Understanding until ~ 2016

\[
\alpha \gtrsim \mathcal{O}(0.1) : \text{Huge energy release} \quad \rightarrow \quad \text{Runaway}
\]
- Plasma friction cannot balance with pressure
- Walls approach the speed of light
- Energy accumulates in \textit{walls}

\[
\alpha \lesssim \mathcal{O}(0.1) : \text{Small energy release} \quad \rightarrow \quad \text{Terminal velocity}
\]
- Plasma friction gets balanced with pressure
- Walls approach terminal velocity
- Energy accumulates in \textit{plasma bulk motion}

[to experts: this is detonation case]

[e.g. Bodeker & Moore, JCAP 0905 (2009) 009
Espinosa et al., JCAP 1006 (2010) 028]
Understanding from 2017 ~

- Plasma friction can balance with pressure
- Walls approach high terminal velocity
- Energy accumulates in plasma bulk motion

- Plasma friction gets balanced with pressure
- Walls approach terminal velocity
- Energy accumulates in plasma bulk motion

[Beheshti & Moore '17]
BEHAVIOR AFTER COLLISION

- What happens after collisions?

1. Walls (energetically subdominant) collide and damp soon
   “bubble collision”

2. Plasma bulk motion continues to propagate
   “sound wave”

3. At late times,
   sound waves develop into nonlinear regime
   “turbulence”
What happens after collisions?

BEHAVIOR AFTER COLLISION

1. Walls (energetically subdominant)
2. Plasma bulk motion continues to propagate "sound wave"
3. At late times, sound waves develop into nonlinear regime "turbulence"

"bubble collision"

[Kosowski et al. '93]
What happens after collisions?

1. Walls (energetically subdominant)
2. Plasma bulk motion continues to propagate
   - "sound wave"

(1) Dynamics is linear for small energy release:
\[
(\partial_t^2 - c_s^2 \nabla^2) u^i = 0 \quad u^i : \text{fluid velocity field}
\]

(2) Width remains to be constant after collision

[Hindmarsh et al. '15]
Following two exhaust $T(x)T(y) \neq 0$ possibilities  

- 1. single-bubble  
- 2. double-bubble  

nucleation point

[Jinno & Takimoto '16]
PHYSICAL INTERPRETATION

- Small wavenumbers sourced at late times

- Wavenumber $k$ sourced when the typical bubble size grows to $\sim 1/k$
GW sourcing as a function of time

- Single

- Double
WHY SINGLE-BUBBLE MATTERS

- Two bubble-wall fragments must remain uncollided until they reach x and y
- Other parts of the bubble might have collided already
- In this sense, breaking of spherical sym. is automatically taken into account