CASE WESTERN RESERVE UNIVERSITY 1826

Kenyon College

Cosmological Observables beyond Linear Gravity

Tom Giblin
August 9, 2017
TeVPA
Columbus, Ohio

1511.01105, 1511.01106,
1608.04403, 1707.06640

work done with James Mertens
and Glenn Starkman

Shouldn't GR be important for the Universe?

Tom Giblin
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The Cosmological Principle

- The Universe is Homogeneous and Isotropic *on large scales*
- Therefore the energy density ad pressure are functions of *time only*

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

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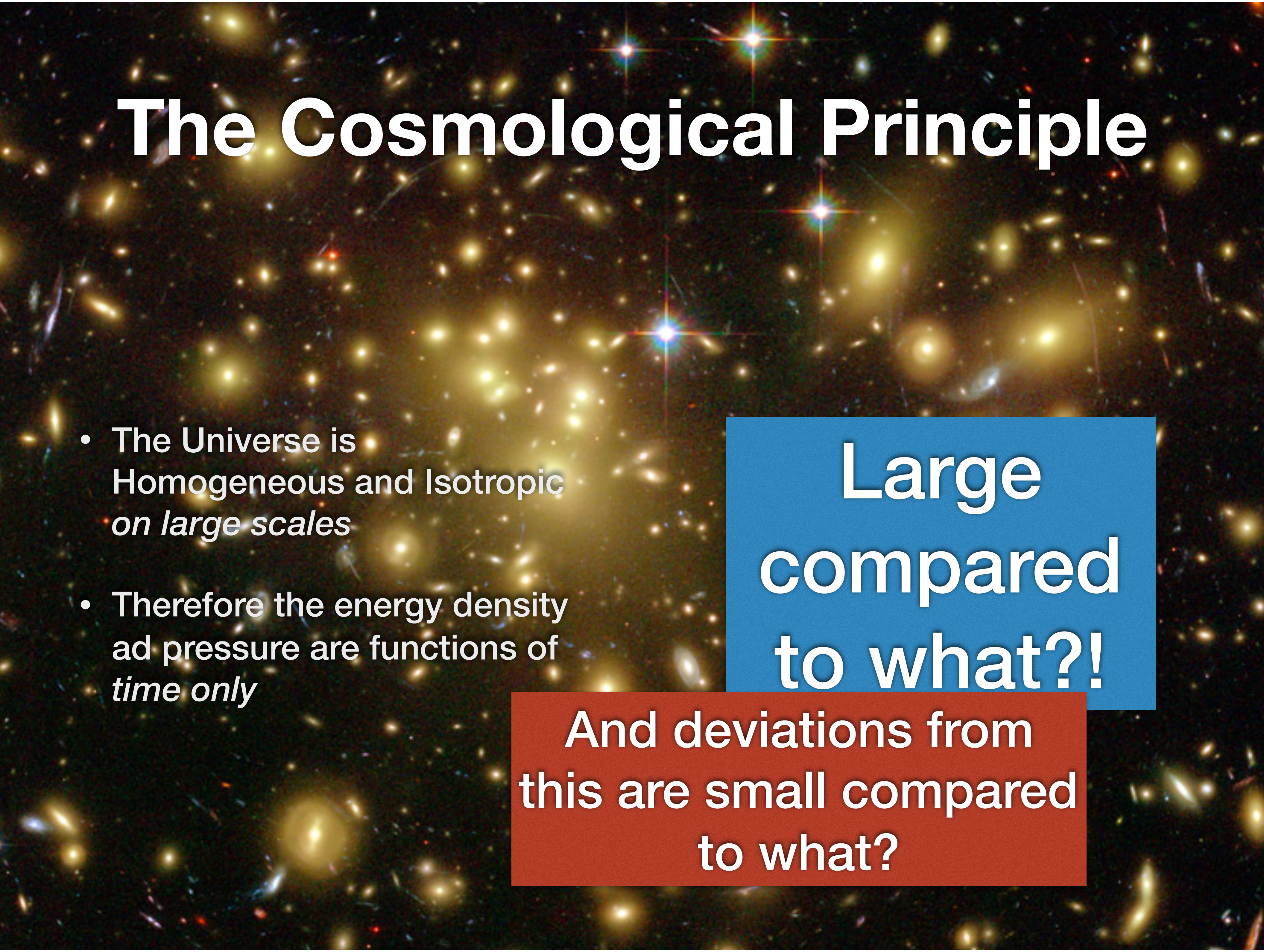
Large compared to what?!



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Large compared to what?!

And deviations from this are small compared to what?

Gravity is Non-Linear

- We like to *separate scales* when doing physics problems (e.g. what happens here, stays here)
- Non-linear physics can mix up scales - power transferred between scales is often referred to as *cascades* or *inverse-cascades*
- The Averaging Problem : When we talk about the expansion of the Universe on the largest of scales, is there *any contribution* from smaller scales?

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1506.06452

Comments on Backreaction

Stephen R. Green[1] **and Robert M. Wald**[2]

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Abstract. We respond to the criticisms of a recent paper of Buchert et al.

Can all cosmological observations be accurately interpreted with a unique geometry?

Pierre Fleury[1,2,*] Hélène Dupuy[1,2,3,†] and Jean-Philippe Uzan[1,2‡]

[1] *Institut d'Astrophysique de Paris, UMR-7095 du CNRS, Université Pierre et Marie Curie, 98 bis bd Arago, 75014 Paris, France.*
[2] *Sorbonne Universités, Institut Lagrange de Paris, 98 bis bd Arago, 75014 Paris, France.*
[3] *Institut de Physique Théorique, CEA, IPhT, URA 2306 CNRS, F-91191 Gif-sur-Yvette, France.*
(Dated: September 4, 2013)

The recent analysis of the Planck results reveals a tension between the best fits for (Ω_{m0}, H_0) derived from the cosmic microwave background or baryonic acoustic oscillations on the one hand, and the Hubble diagram on the other hand. These observations probe the universe on very different scales since they involve light beams of very different angular sizes, hence the tension between them may indicate that they should not be interpreted the same way. More precisely, this letter questions the accuracy of using only the (perturbed) Friedmann-Lemaître geometry to interpret all the cosmological observations, regardless of their angular or spatial resolution. We show that using an inhomogeneous "Swiss-cheese" model to interpret the Hubble diagram allows us to reconcile the inferred value of Ω_{m0} with the Planck results. Such an approach does not require us to invoke new physics nor to violate the Copernican principle.

1304.7791

Gravity is Non-Linear

- W … cs
  p …
- N …
  tr …
  c …
- The Averaging … he
  expansion of t … les, is
  there *any cont* …

Concordance cosmology without dark energy

Gábor Rácz[1★], László Dobos[1], Róbert Beck[1], István Szapudi[2], István Csabai[1]
[1] *Department of Physics of Complex Systems, Eötvös Loránd University, Pf. 32, H-1518 Budapest, Hungary*
[2] *Institute for Astronomy, University of Hawaii, 2680 Woodlawn Drive, Honolulu, HI, 96822*

Accepted XXX. Received YYY; in original form ZZZ

1607.08797

ABSTRACT

According to the separate universe conjecture, spherically symmetric sub-regions in an isotropic universe behave like mini-universes with their own cosmological parameters. This is an excellent approximation in both Newtonian and general relativistic theories. We estimate local expansion rates for a large number of such regions, and use a scale parameter calculated from the volume-averaged increments of local scale

1506.06452

…ld[2]

…Y5, Canada
…, The University of Chicago
…SA
…@uchicago.edu

…t paper of Buchert et al.

…ry?

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1304.7791

Averaging

- Generally a Hubble Volume is taken to be the region over which we do averaging — we all agree that different Hubble patches could have different expansion rates (causality, right?)

$$H^{-3} \approx (4000\,\mathrm{Mpc})^3$$

- Yet there is structure at (just) smaller scales
  - Galaxy Clusters $\sim 1 - 10\,\mathrm{Mpc}$
  - Inter-Cluster Distances $\sim 50\,\mathrm{Mpc}$

What you would *like* to do

- Write down the most general form of the metric,

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{01} & g_{11} & g_{12} & g_{13} \\ g_{02} & g_{12} & g_{22} & g_{23} \\ g_{03} & g_{13} & g_{23} & g_{33} \end{pmatrix}$$

- Plug it into Einstein's Equations

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

- Solve the system of second order differential equations (subject to your gauge-constraints)

```
In[9]:= SetDirectory[NotebookDirectory[]];

In[10]:= << GREAT.m

GREAT functions are: IMetric, Christoffel,
  Riemann, Ricci, SCurvature, EinsteinTensor, SqRicci, SqRiemann.

Enter 'helpGREAT' for this list of functions

In[11]:= (metric = {{g00[x0, x1, x2, x3], g01[x0, x1, x2, x3], g02[x0, x1, x2, x3],
        g03[x0, x1, x2, x3]}, {g01[x0, x1, x2, x3], g11[x0, x1, x2, x3],
        g12[x0, x1, x2, x3], g03[x0, x1, x2, x3]},
       {g02[x0, x1, x2, x3], g12[x0, x1, x2, x3], g22[x0, x1, x2, x3],
        g23[x0, x1, x2, x3]}, {g03[x0, x1, x2, x3], g13[x0, x1, x2, x3],
        g23[x0, x1, x2, x3], g33[x0, x1, x2, x3]}}) // MatrixForm

Out[11]//MatrixForm=
 ( g00[x0, x1, x2, x3]  g01[x0, x1, x2, x3]  g02[x0, x1, x2, x3]  g03[x0, x1, x2, x3] )
 ( g01[x0, x1, x2, x3]  g11[x0, x1, x2, x3]  g12[x0, x1, x2, x3]  g03[x0, x1, x2, x3] )
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In[12]:= coords = {x0, x1, x2, x3}

Out[12]= {x0, x1, x2, x3}

In[13]:= EinsteinTensor[metric, coords]
```

What can we do?

- You can do a little better by making gauge choices that reduce the number of parameters or (re)parameterize so that you have nice equations for.. some.. of them…
- Even then they are extremely difficult to numerically stabilize

Numerical Relativity and Compact Binaries

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[b] *Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61820*

[c] *Department of Astronomy and NCSA, University of Illinois at Urbana-Champaign, Urbana, IL 61820*

Abstract

Numerical relativity is the most promising tool for theoretically modeling the inspiral and coalescence of neutron star and black hole binaries, which, in turn, are among the most promising sources of gravitational radiation for future detection by gravitational wave observatories. In this article we review numerical relativity approaches to modeling compact binaries. Starting with a brief introduction to the 3+1 decomposition of Einstein's equations, we discuss important components of numerical relativity, including the initial data problem, reformulations of Einstein's equations, coordinate conditions, and strategies for locating and handling black holes on numerical grids. We focus on those approaches which currently seem most relevant for the compact binary problem. We then outline how these methods are used to model binary neutron stars and black holes, and review the current status of inspiral and coalescence simulations.

Key words:

Contents

What we have to do…

- Luckily there are a set of new approaches. We use the most common of these: the BSSN formalism.

- It is based on the ADM metric decomposition

$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \gamma_{lk}\beta^l\beta^k & \beta_i \\ \beta_j & \gamma_{ij} \end{pmatrix}$$

- We we introduce more parameters than (minimally) necessary so that the equations are easier to solve

In Cosmology

- We can fix the gauge (we will give up being able to create black holes, as well as some other concessions) to focus on spatial slices

- We can then track the spatial 3-metric

$$\gamma_{ij} = e^{4\phi}\bar{\gamma}_{ij}$$

- as well as the extrinsic curvature

$$K_{ij} = e^{4\phi}\bar{A}_{ij} + \frac{1}{3}\gamma_{ij}K$$

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In Cosmology

- We can fix the gauge (we will give up being able to create black holes, as well as some other concessions) to focus on spatial slices

- We can then track the spatial 3-metric

Think of this as keeping track of the size of local volumes

$$\gamma_{ij} = e^{4\phi}\bar{\gamma}_{ij}$$

- as well as the extrinsic curvature

Think of this as measuring the local expansion rate

$$K_{ij} = e^{4\phi}\bar{A}_{ij} + \frac{1}{3}\gamma_{ij}K$$

Importantly

These variables have well-behaved differential equations and *are a complete description of GR without additional constraints*

$$\partial_t \phi = -\frac{1}{6} K$$

$$\partial_t \bar{\gamma}_{ij} = -2\bar{A}_{ij}$$

$$\partial_t K = \bar{A}_{ij}\bar{A}^{ij} + \frac{1}{3}K^2 + 4\pi(\rho + S)$$

$$\partial_t \bar{A}_{ij} = e^{-4\phi}(R_{ij} - 8\pi S_{ij})^{TF} + K\bar{A}_{ij} - 2\bar{A}_{il}\bar{A}^l_j$$

$$\partial_t \bar{\Gamma}^i = 2\bar{\Gamma}^i_{jk}\bar{A}^{jk} - \frac{4}{3}\bar{\gamma}^{ij}\partial_j K - 16\pi\bar{\gamma}^{ij}S_j + 12\bar{A}^{ij}\partial_j \phi \,.$$

Importantly

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$$\partial_t \phi = -\frac{1}{6} K$$

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We chose synchronous gauge (cosmology) / geodesic slicing (Numerical GR)

$$\alpha = 1,\ \beta^i = 0$$

With a Source

- As a first-guess; we take a Universe to be filled with a pressureless, non-interacting* perfect fluid with

$$w = 0$$

- This fluid obeys a fluid equation,

$$\partial_t \tilde{D} = \partial_t(\gamma^{1/2}\rho_0) = 0$$

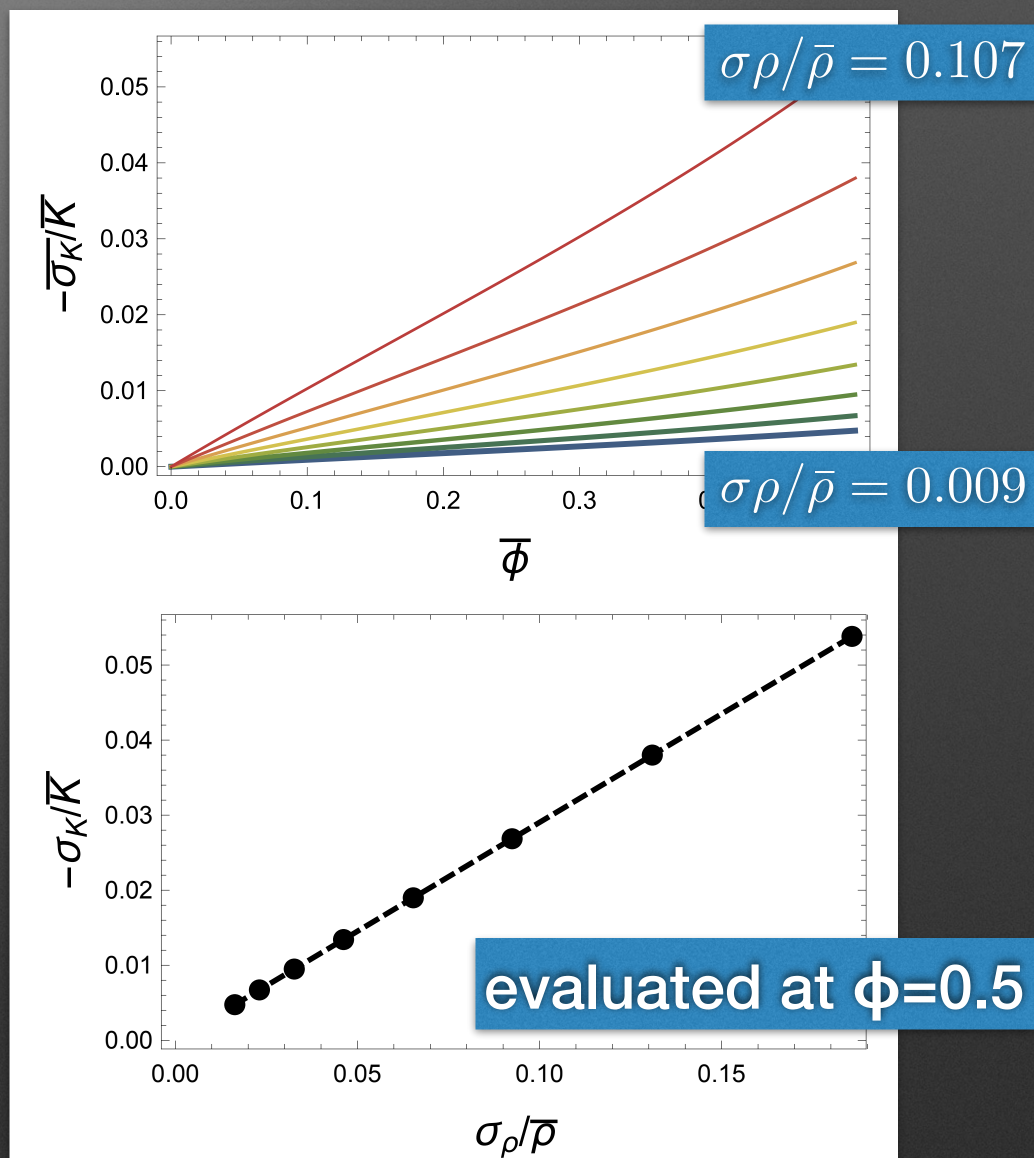
- which vanishes in *synchronous* gauge. *Therefore the the fluid doesn’t evolve (in our coordinates)

Observables

Hubble Diagrams

The Universe Gets Clumpy

- We can now compare the statistics of *K* as a function of the initial density contrast
- And how that statistic changes in time



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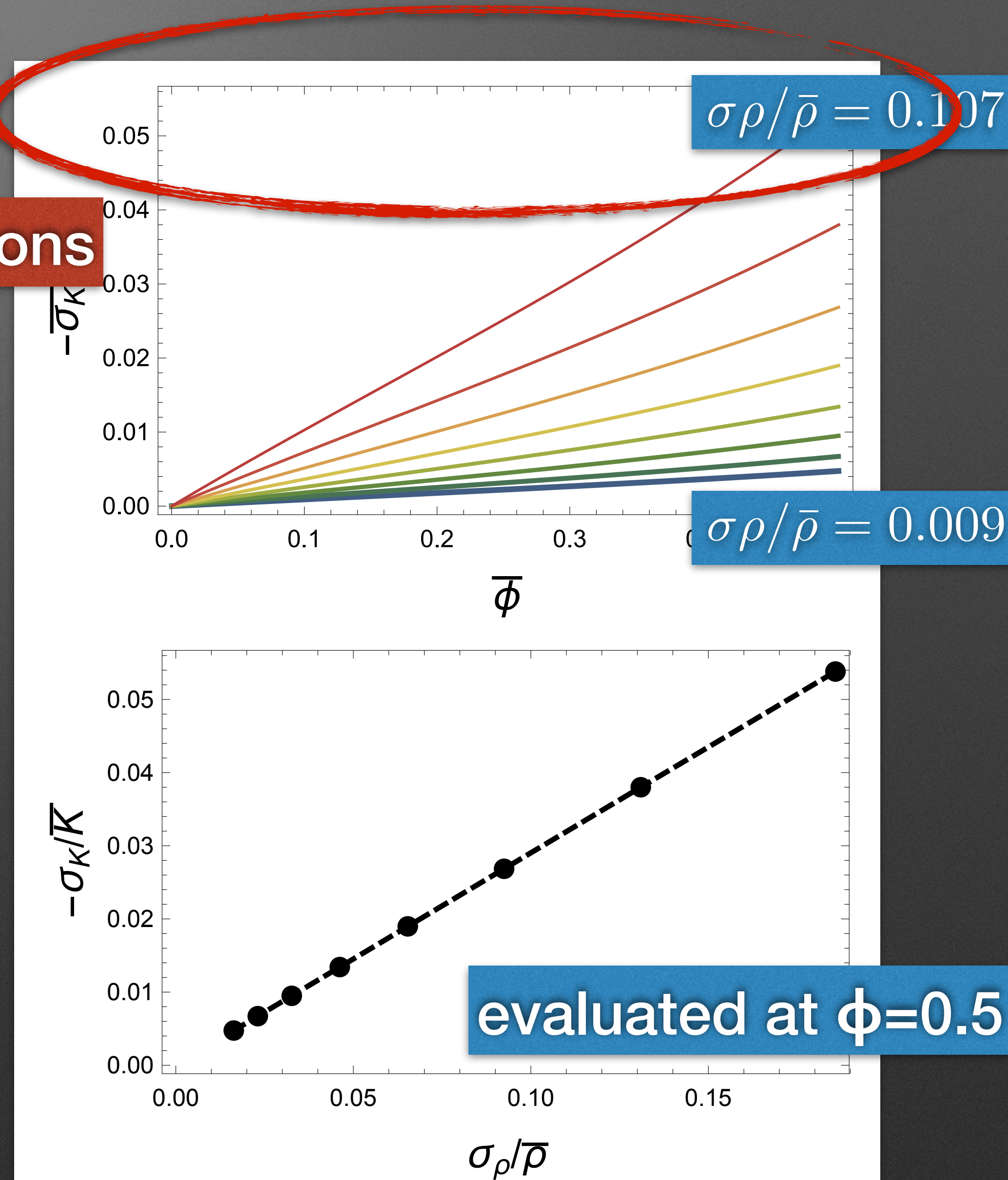
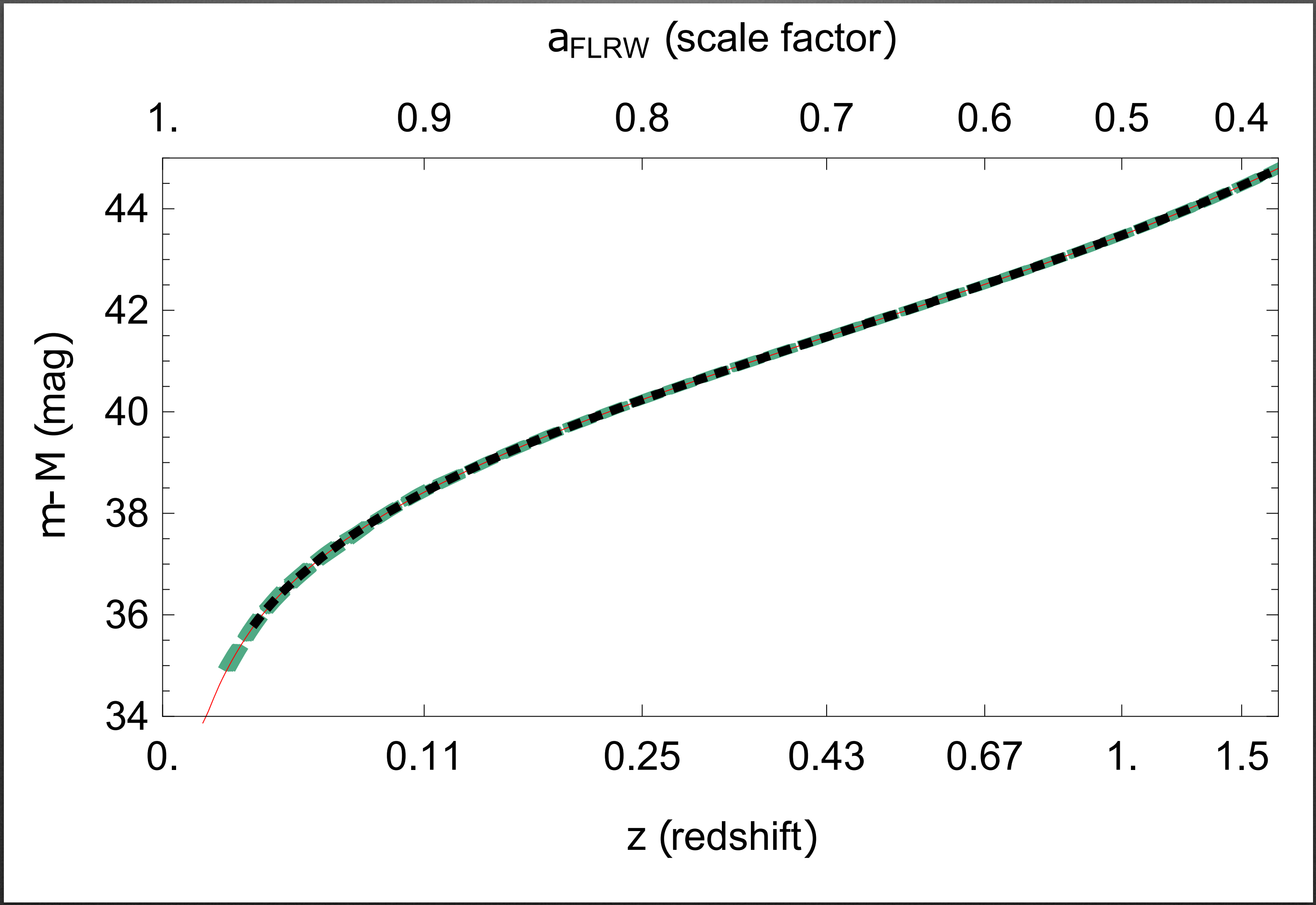


The effect on the Hubble Diagram



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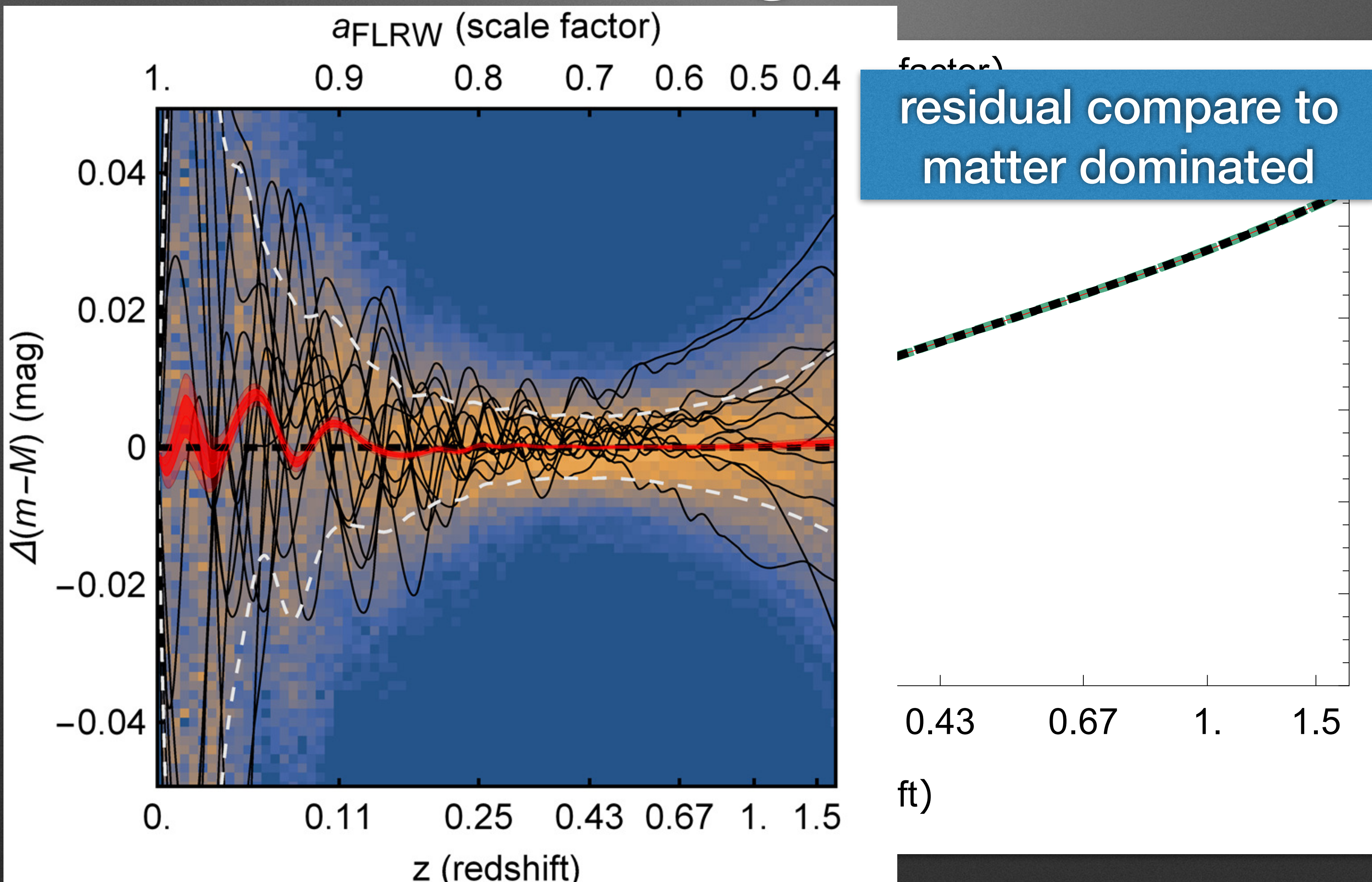

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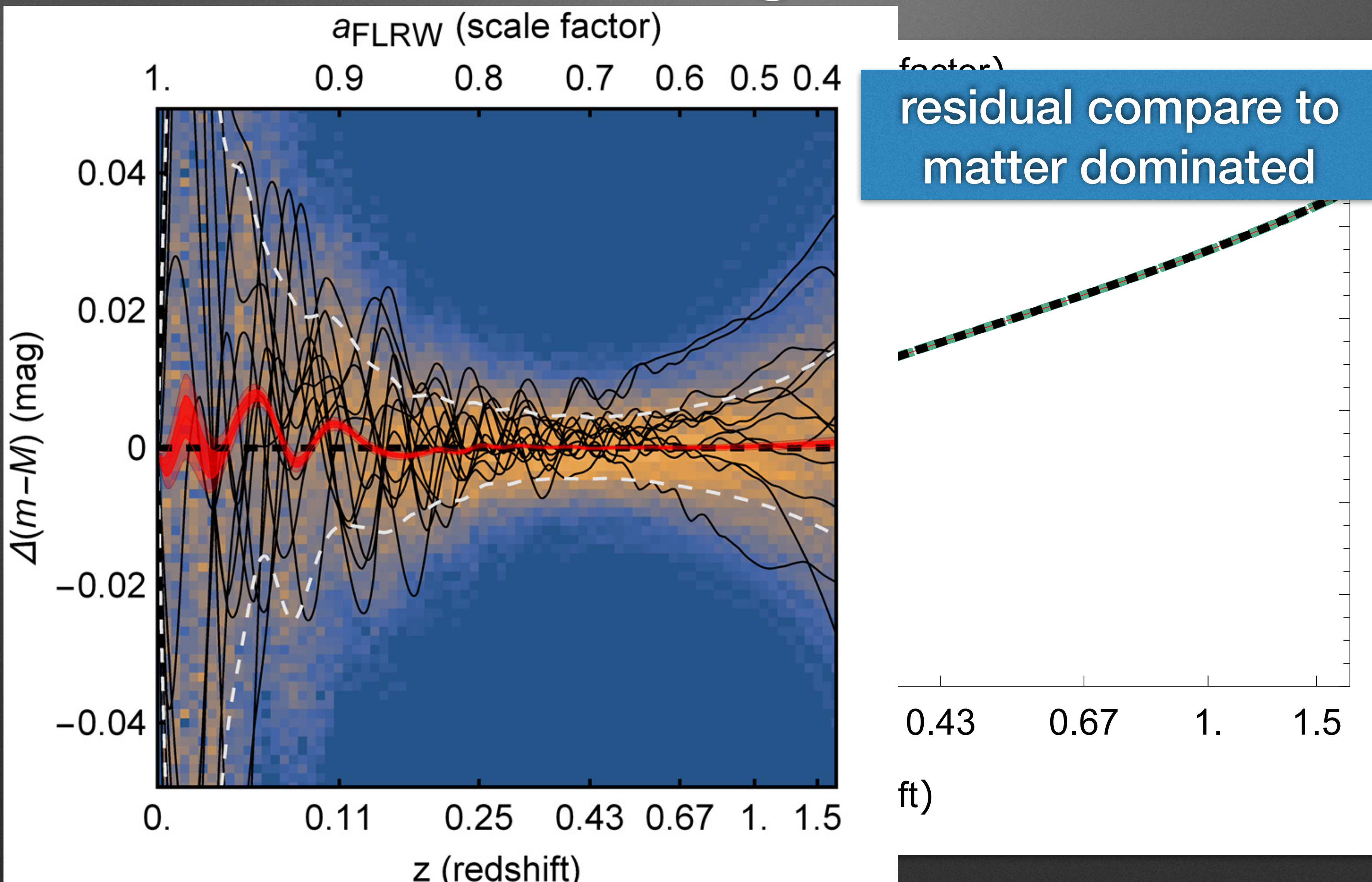

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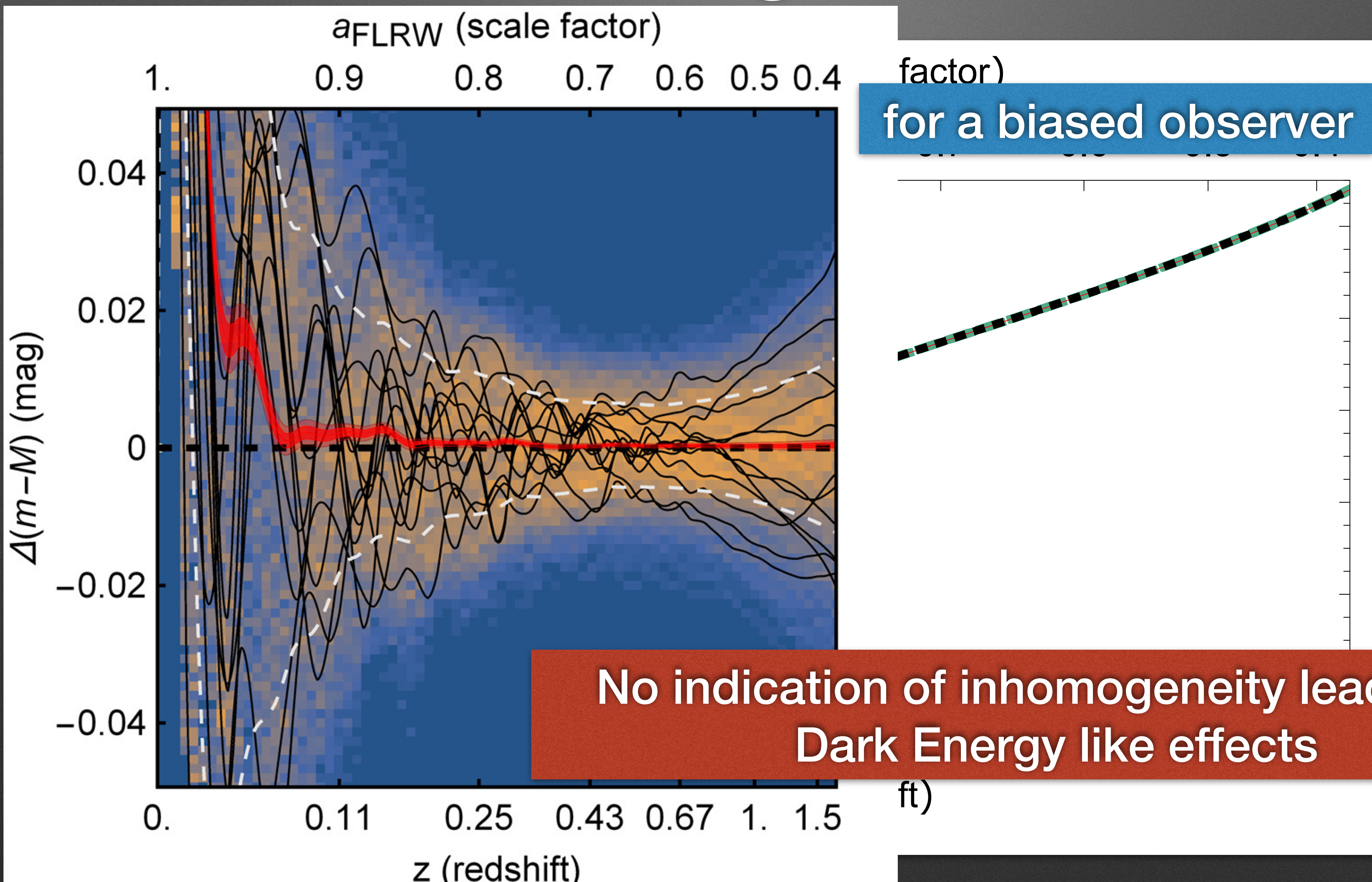


for a biased observer

No indication of inhomogeneity leading to Dark Energy like effects

The Lensing Potential

Comparing Perturbation Theory

- We can *try* to run our BSSN formalism in Newtonian Gauge

$$\alpha \approx 1 + \Phi \qquad\qquad e^{4\phi} \approx a^2(1-\Phi)$$

- Of course, this sets a gauge (slicing) condition

$$\dot{\alpha} = \dot{\Phi} = \frac{K + \bar{K}(\alpha - 2)}{3} \approx \frac{\delta K}{3}$$

- which is *unstable*

Calculate Invariants (To Do…)

- Calculate the Bardeen potentials from the BSSN variables

$$A = \frac{1}{2a^2}\left(h_{ii} - \frac{1}{\nabla^2}\partial_i\partial_j h_{ij}\right)$$

$$h_{ij} = \gamma_{ij} - a^2\delta_{ij}$$

$$\dot{h}_{ij} = -2K_{ij} - 2a\dot{a}\delta_{ij}$$

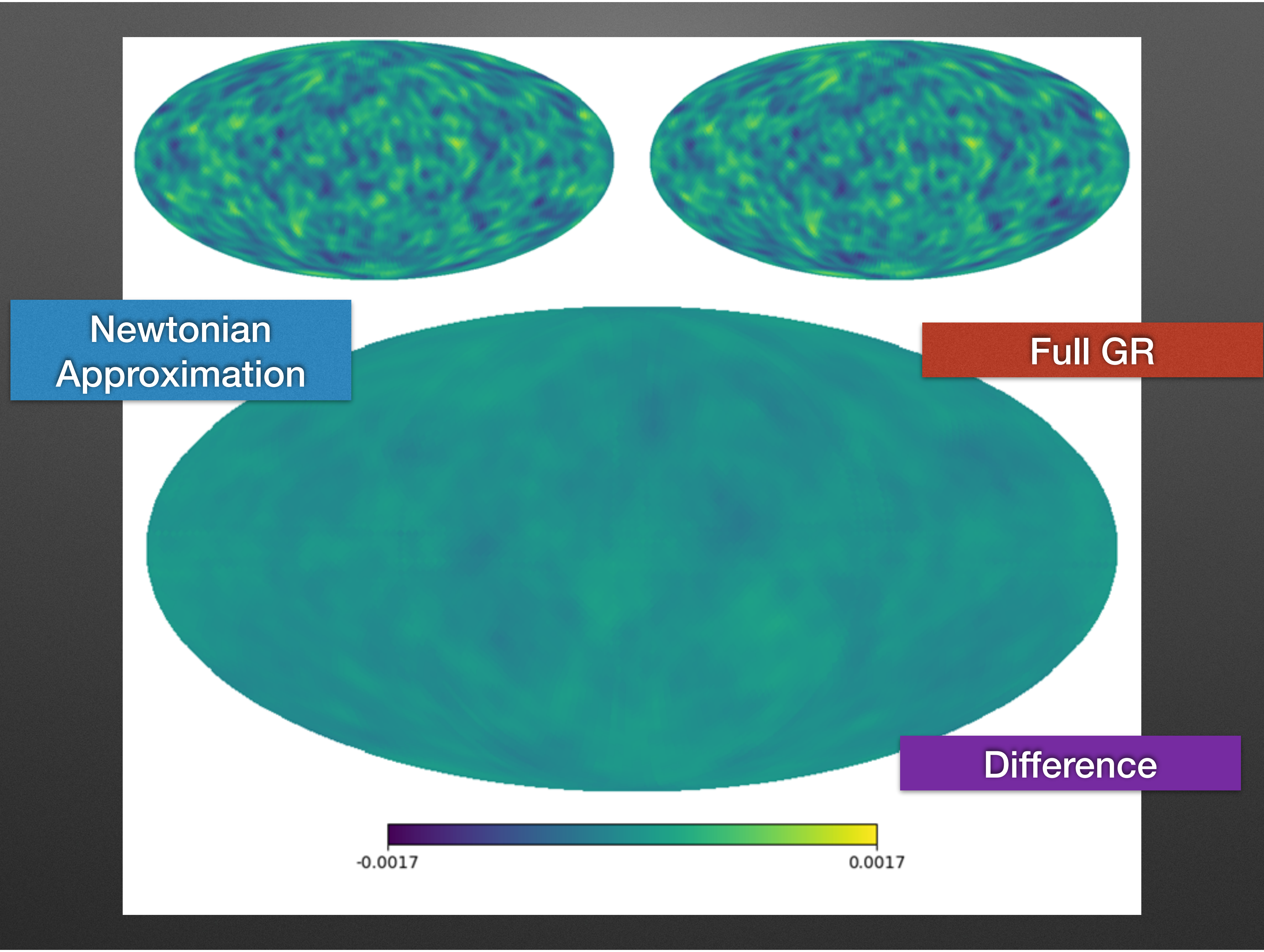
$$B = \frac{1}{\nabla^2}\left(\frac{h_{ii}}{a^2} - 3A\right),$$

$$\ddot{h}_{ij} = -2\left(\dot{K}_{ij} + \left(\dot{a}^2 + a\ddot{a}\right)\delta_{ij}\right)$$

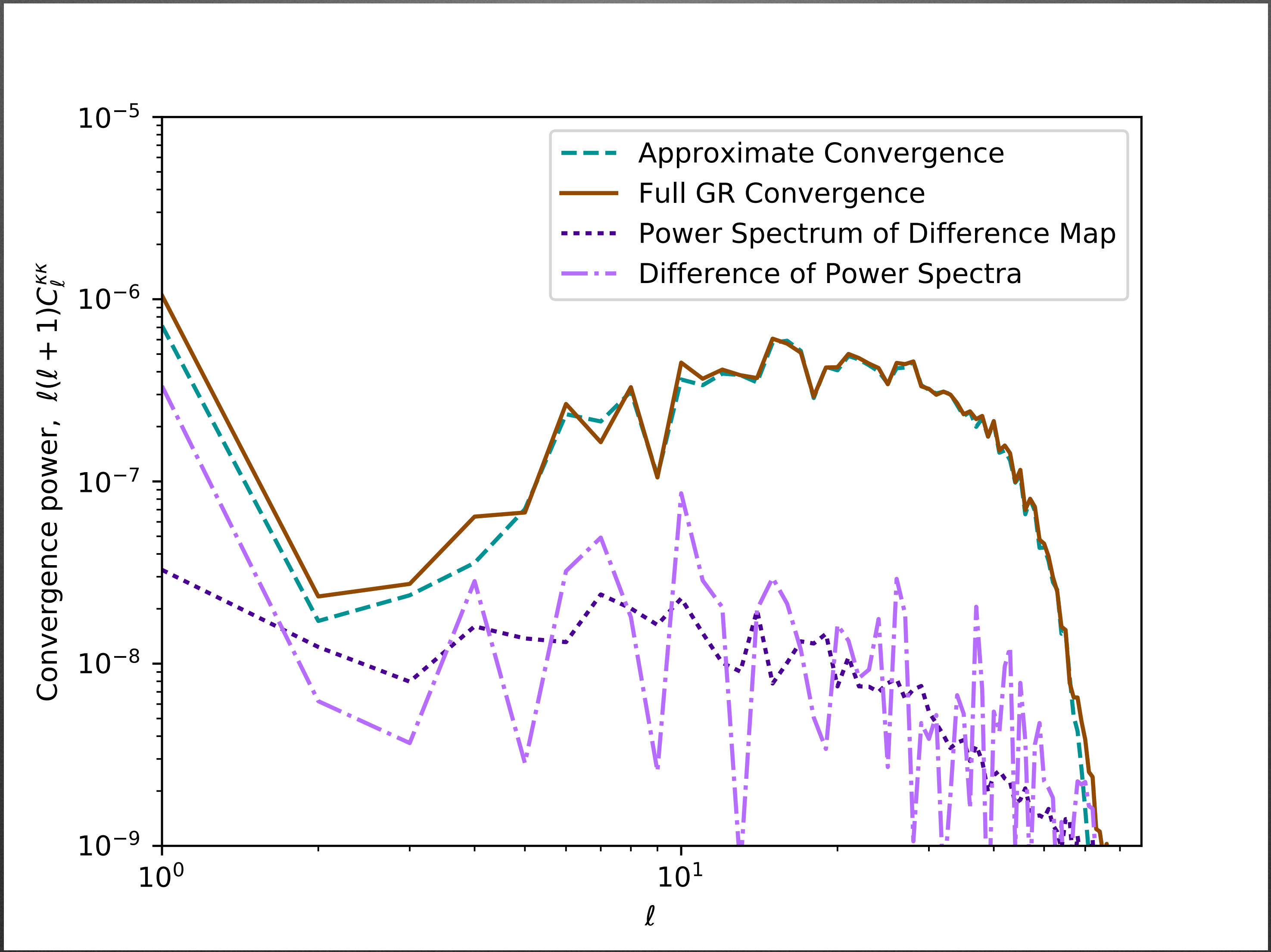
$$\Psi = -\frac{a}{2}\left(2\dot{a}\dot{B} + a\ddot{B}\right)$$

$$\Phi = \frac{1}{2}\left(a\dot{a}\dot{B} - A\right)$$

- Calculate Weyl Scalars from both models.


Newtonian Approximation
Full GR
Difference
-0.0017
0.0017

Power Spectrum



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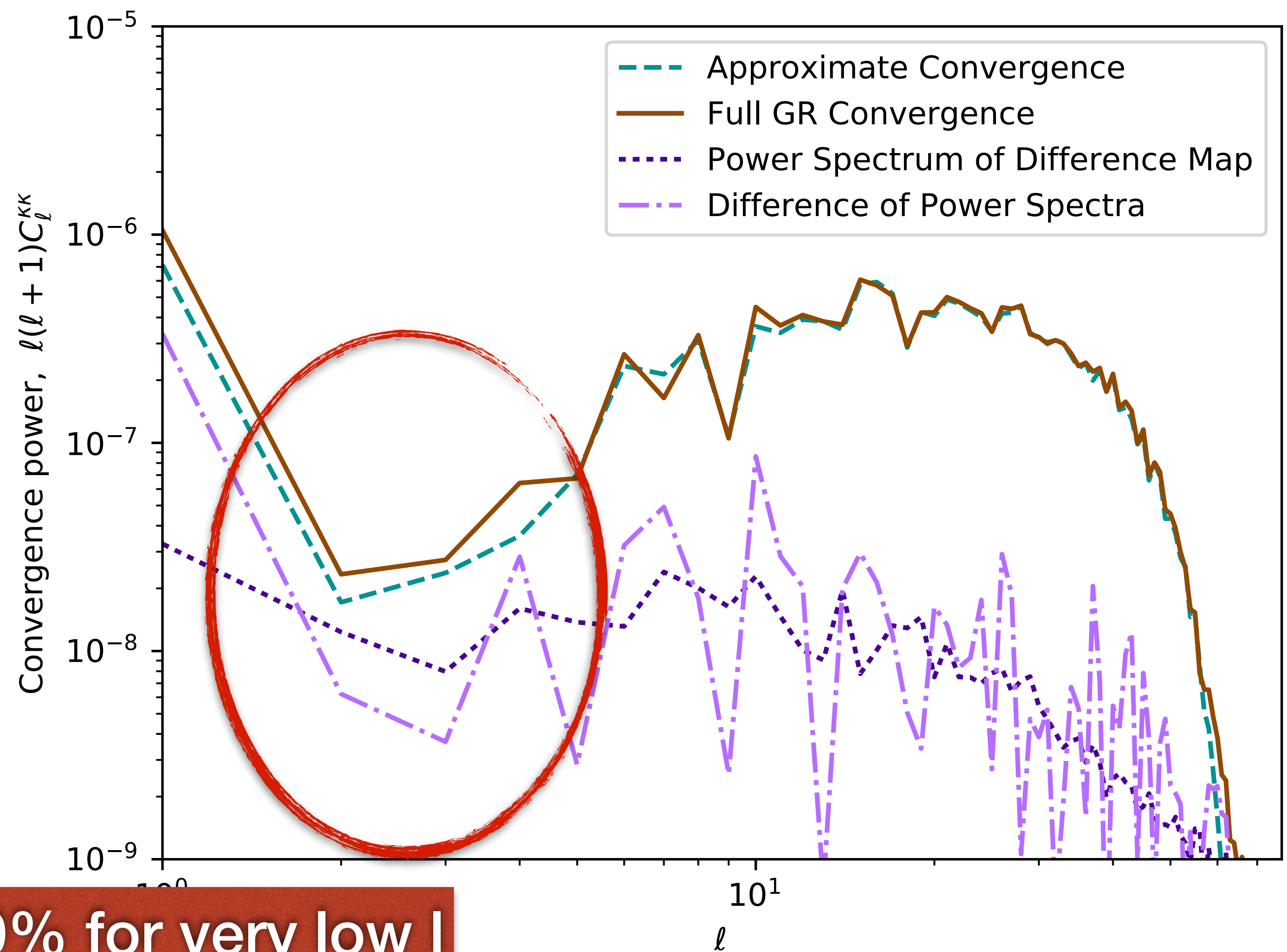


~10% for very low l

Your Take-home

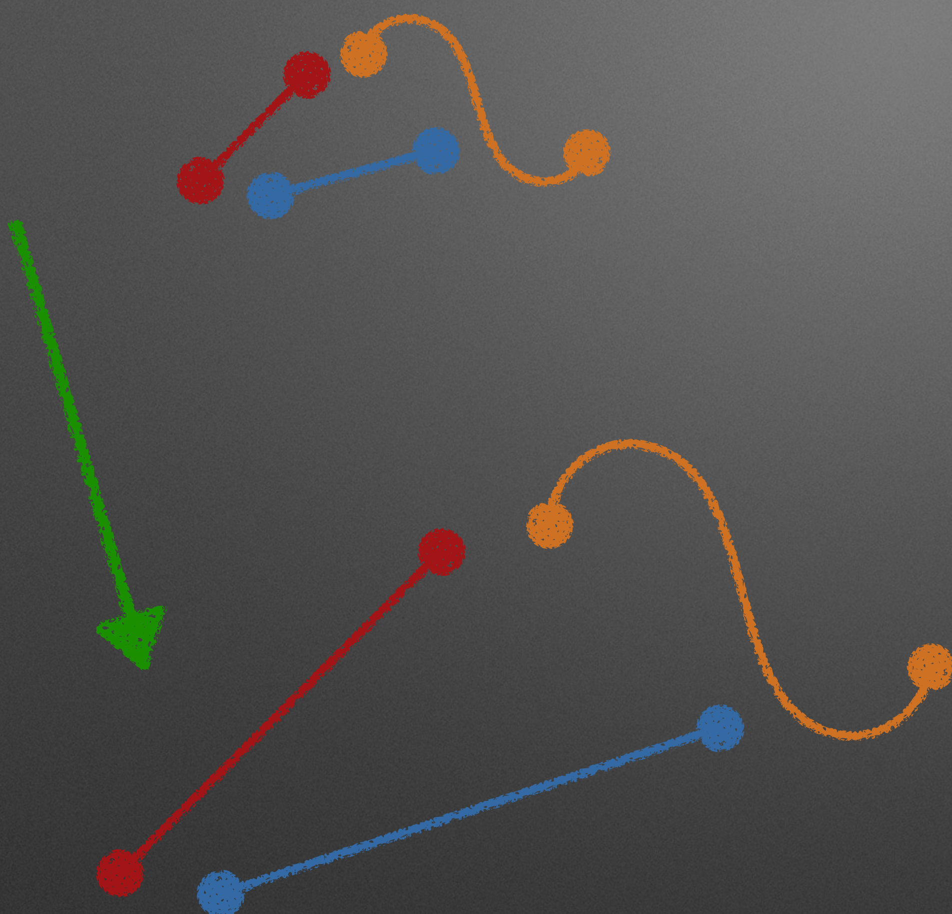
- There are (yet) not paradigmatic changes due to full Numerical Relativity
- Full Relativistic effects are 1-10 % level modifications to precision observables

Fin

Question #3

How would *an* observer know?

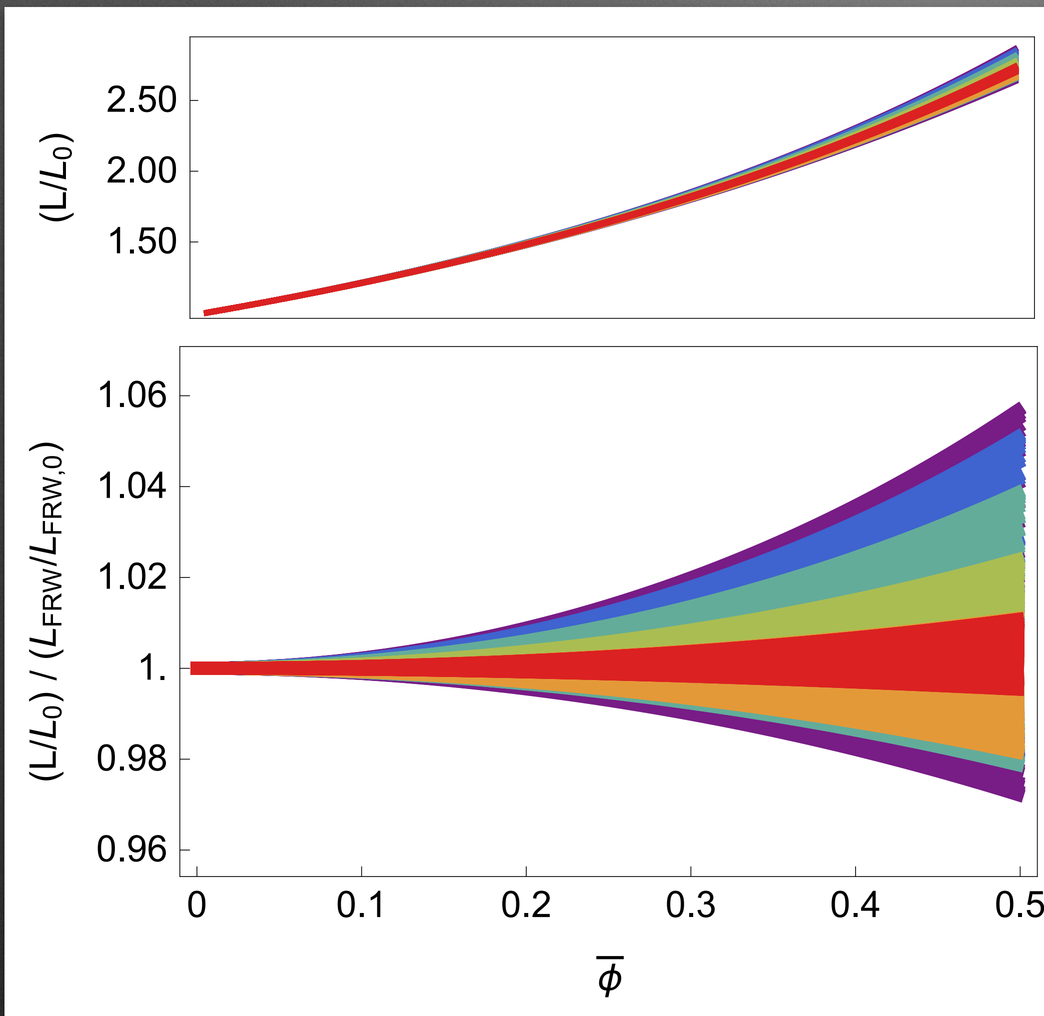
Compare to FLRW



we define a lot of paths along coordinate axes (of different lengths) and see how their proper length evolves over time

- In a homogeneous space (where the *expansion rate* is constant over the spatial slices) we expect the *proper length* of paths to scale with the scale factor

Compare to FLRW



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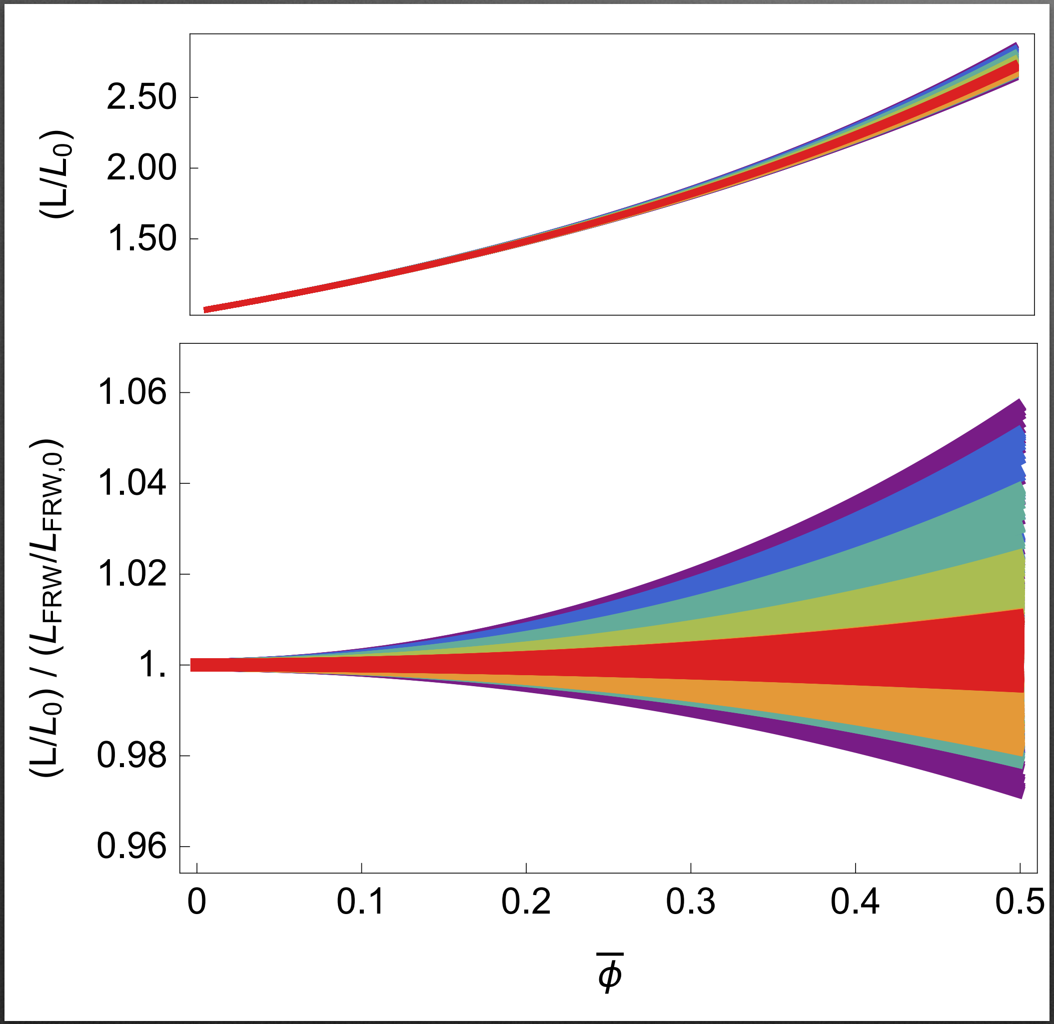
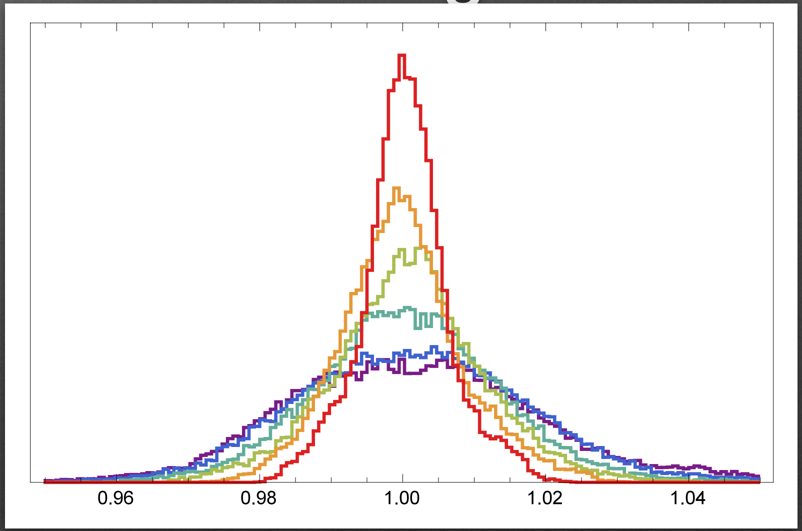
Compare to FLRW



- In a homogeneous



Constructing Null Geodesics

- We start with the geodesic equation

$$\frac{d^2x^\mu}{d\lambda^2} = -\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda}$$

- recast in terms of the independent variable (of the code)

$$\frac{\mathrm{d}^2X^\mu}{\mathrm{d}t^2} = -\frac{d^2x^\mu}{d\lambda^2} = -\Gamma^\mu_{\alpha\beta}\frac{\mathrm{d}X^\alpha}{\mathrm{d}t}\frac{\mathrm{d}X^\beta}{\mathrm{d}t} + \Gamma^0_{\alpha\beta}\frac{\mathrm{d}X^\alpha}{\mathrm{d}t}\frac{\mathrm{d}X^\beta}{\mathrm{d}t}\frac{\mathrm{d}X^\mu}{\mathrm{d}t}$$

- where we will define

$$q^\mu = \frac{dx^\mu}{dt} = \alpha(n^\mu + V^\mu) \quad \text{and} \quad p^\mu = E(n^\mu + V^\mu)$$

Which gives us a set of equations to solve....

$$\frac{\mathrm{d}X^i}{\mathrm{d}t} = \alpha V^i - \beta^i$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = E\left(\alpha K_{ij} V^i V^j - V^j \partial_j \alpha\right)$$

$$\frac{\mathrm{d}V^i}{\mathrm{d}t} = \alpha V^j \left(V^i \partial_j \ln\alpha - K_{jk} V^k V^i + 2K^i_j - {}^{(3)}\Gamma^i_{jk} V^k\right) - \gamma^{ij}\partial_j\alpha - V^j \partial_j \beta^i$$

- Which needs to be solved along a set of trajectories
- We don’t know where we end up (only where they start)
- And they don’t lie *on* lattice points.

No Problem

- We start an large number (500) in arbitrary positions, and in arbitrary directions
- We interpolate the fields along the paths (the lattice points are pretty close together)
- At the end of the simulation we can look at the histories of the particles and draw Hubble Diagrams

No Problem

well…for Jim

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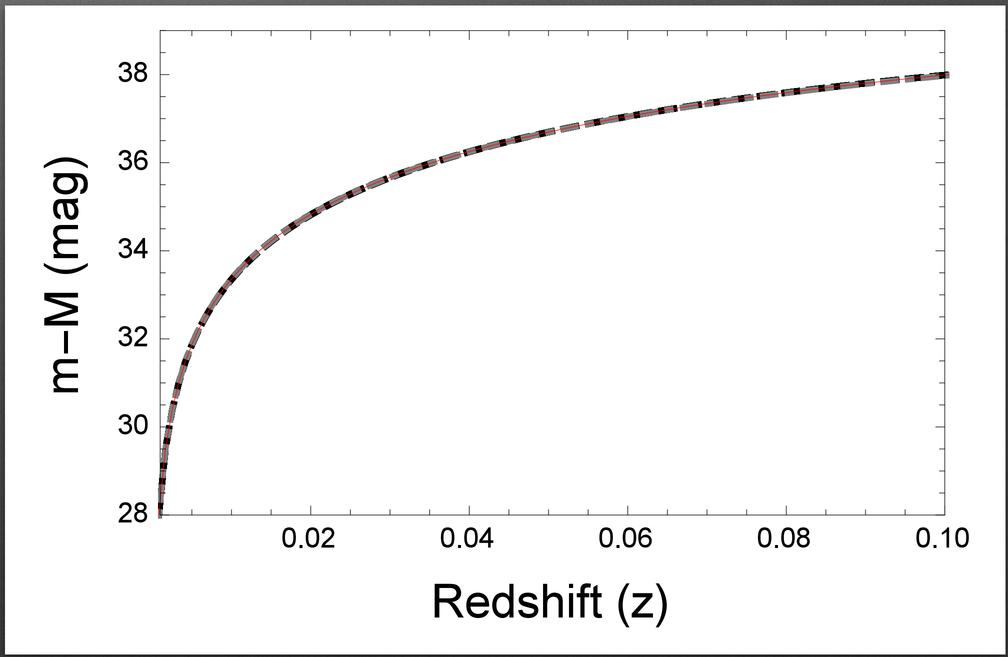
- We interpolate the fields along the paths (the lattice points are pretty close together)
- At the end of the simulation we can look at the histories of the particles and draw Hubble Diagrams

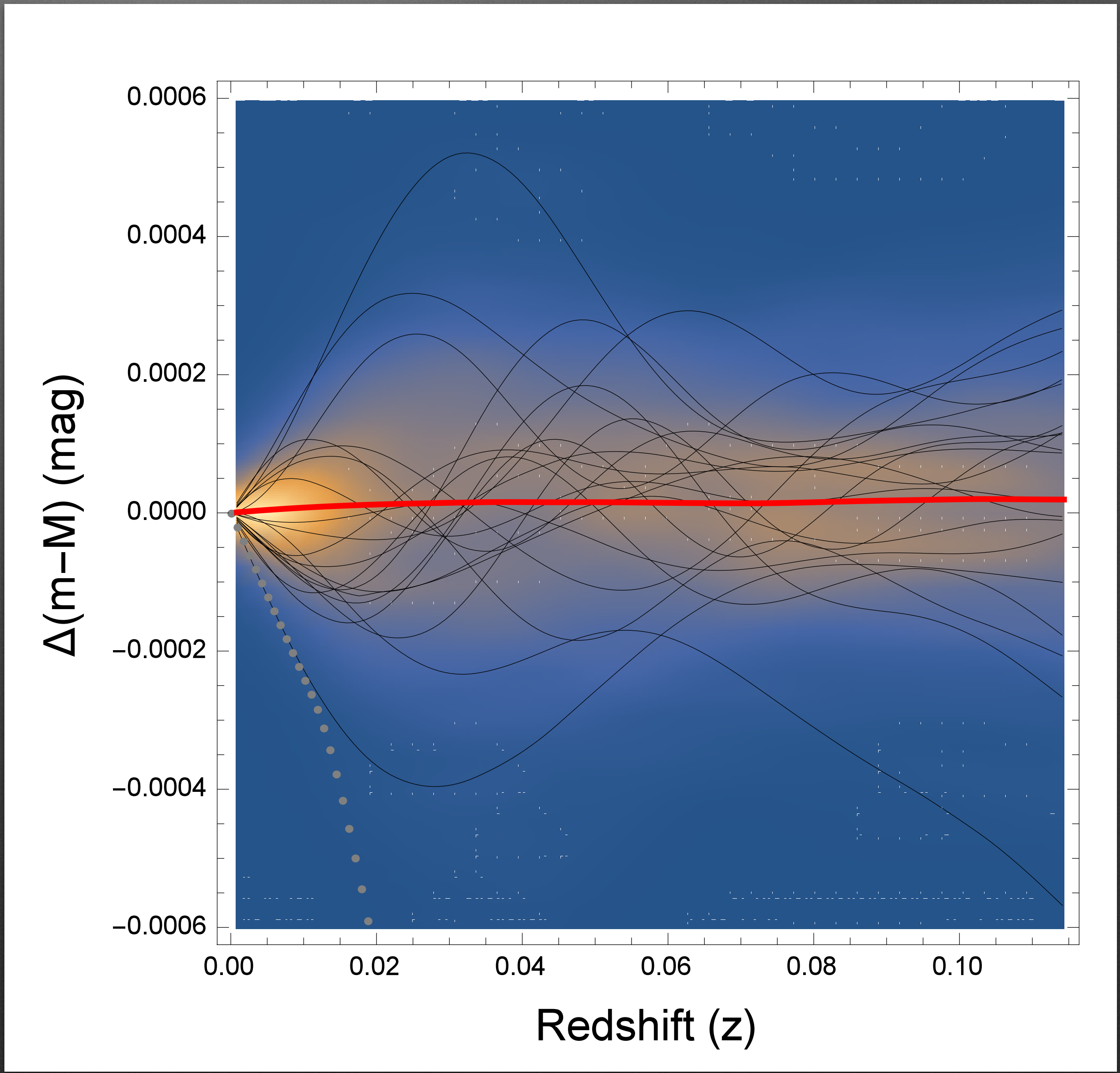
Averaged Observers

- Good News:
  - Almost indistinguishable agreement with LCDM (and with ΩM=1)
- Bad News:
  - Only redshift of 0.1…



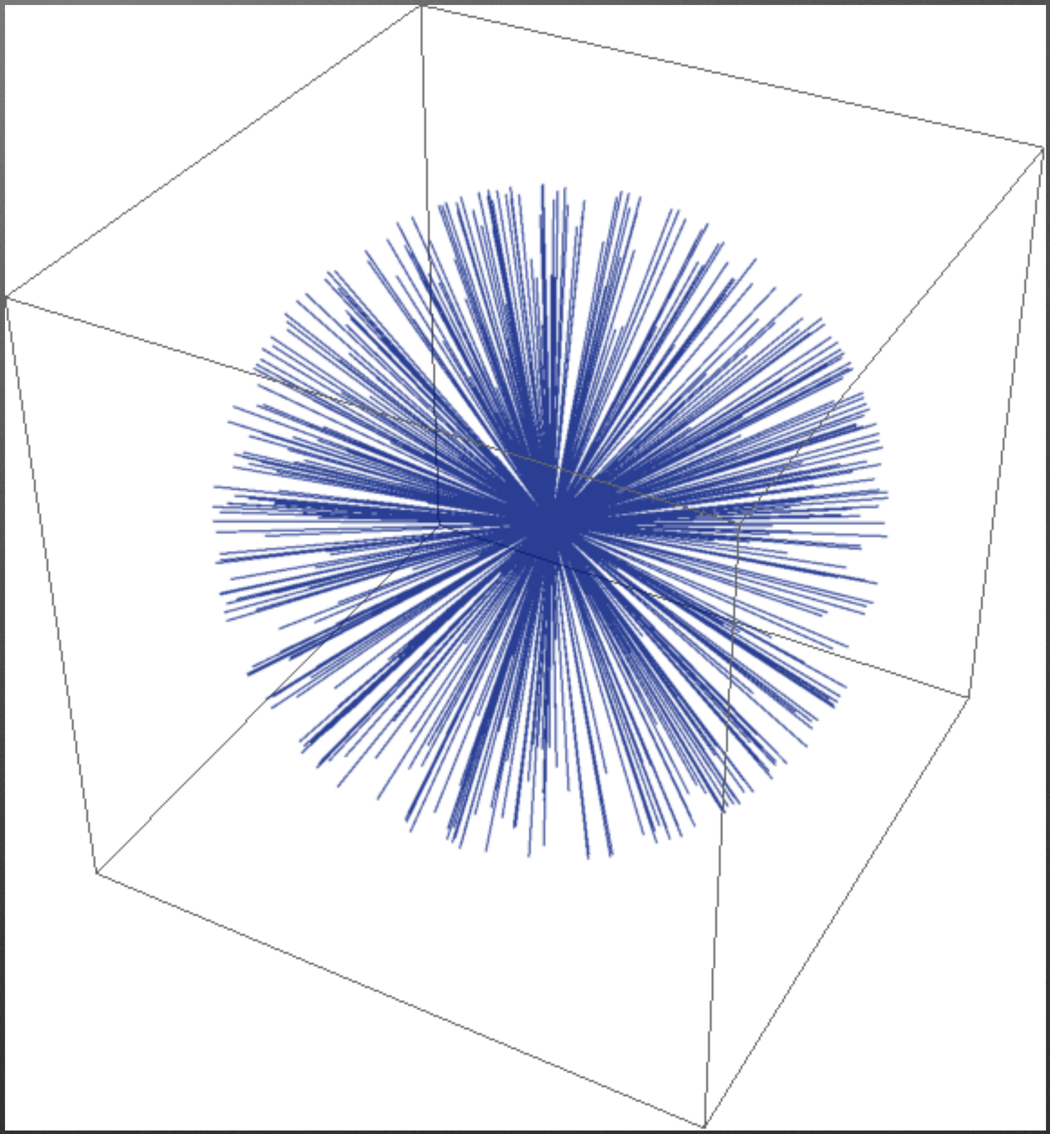
We look at the residuals

- If we look at the residuals we see that an *averaged* observer see a matter dominated Hubble diagram



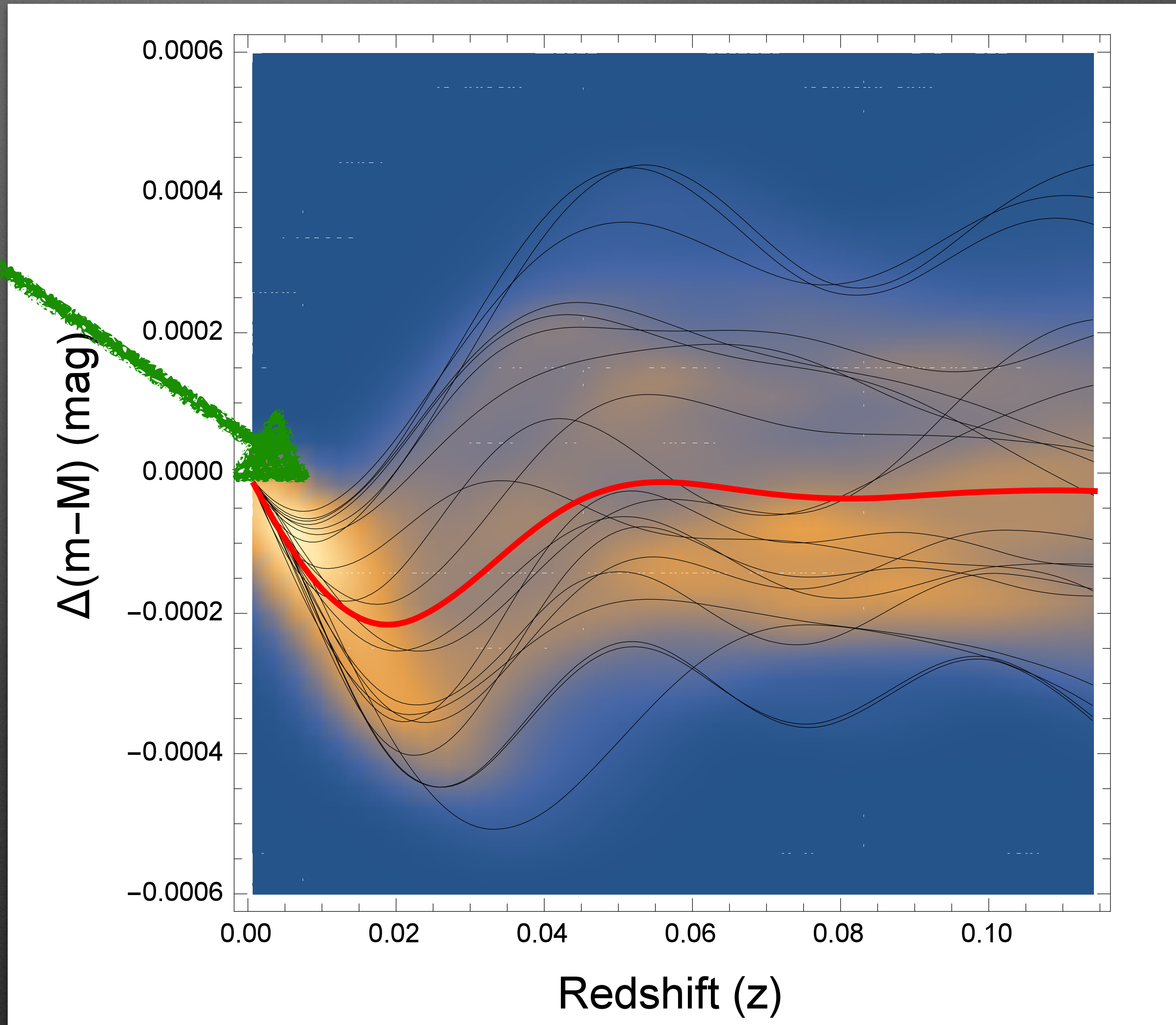
Biased Observer (kinda)

- The deviations from "straight" aren't huge for this toy Universe
- So we take a set of points that we know will end up at (approximately) the same location
- Which is an over density of about 10%



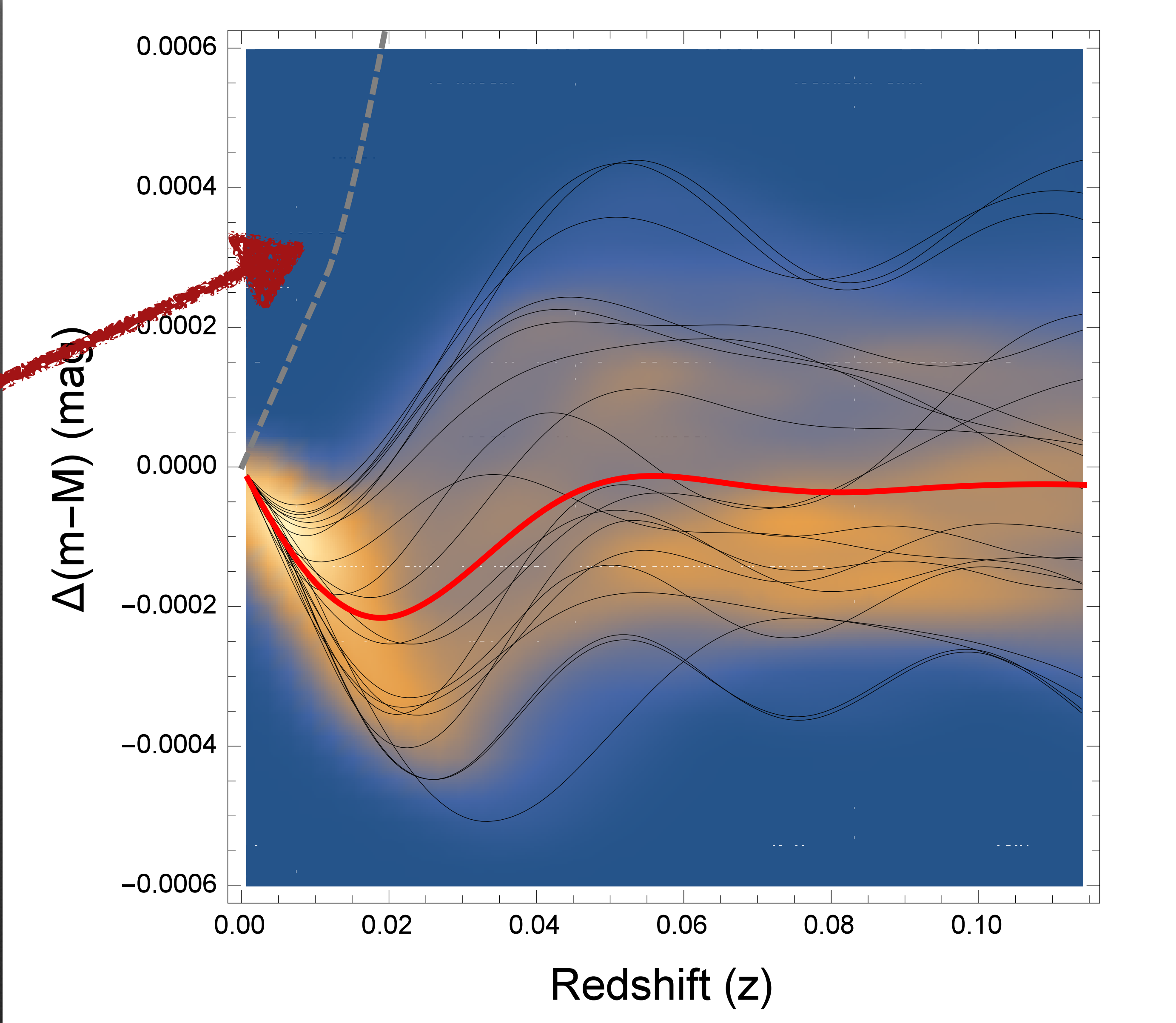
And the Residuals are...

We see a bias at low z



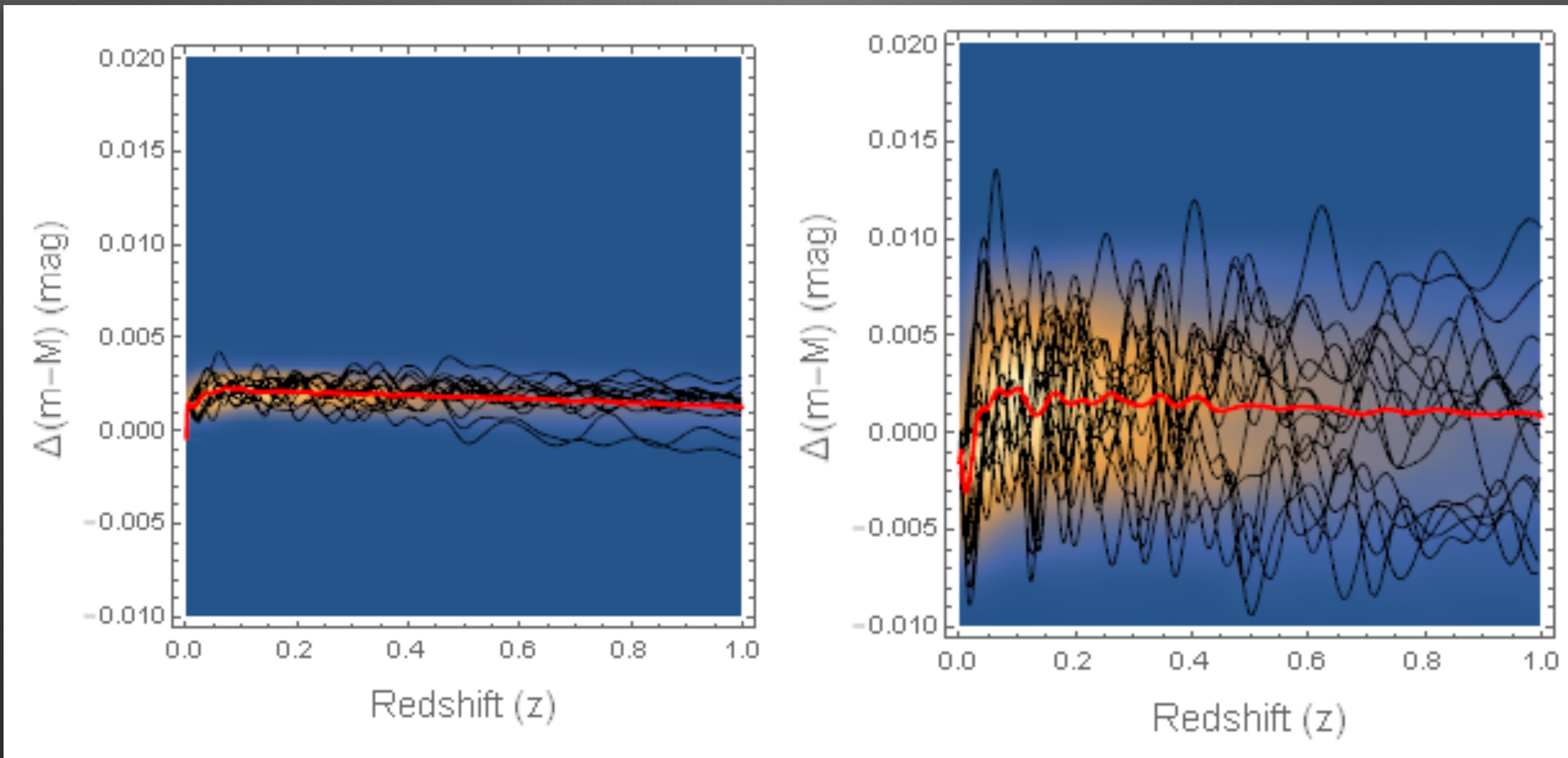
And the Residuals are…

but no indications (yet) that this mimics LCDM



Very Recently:
We can run the program backward.

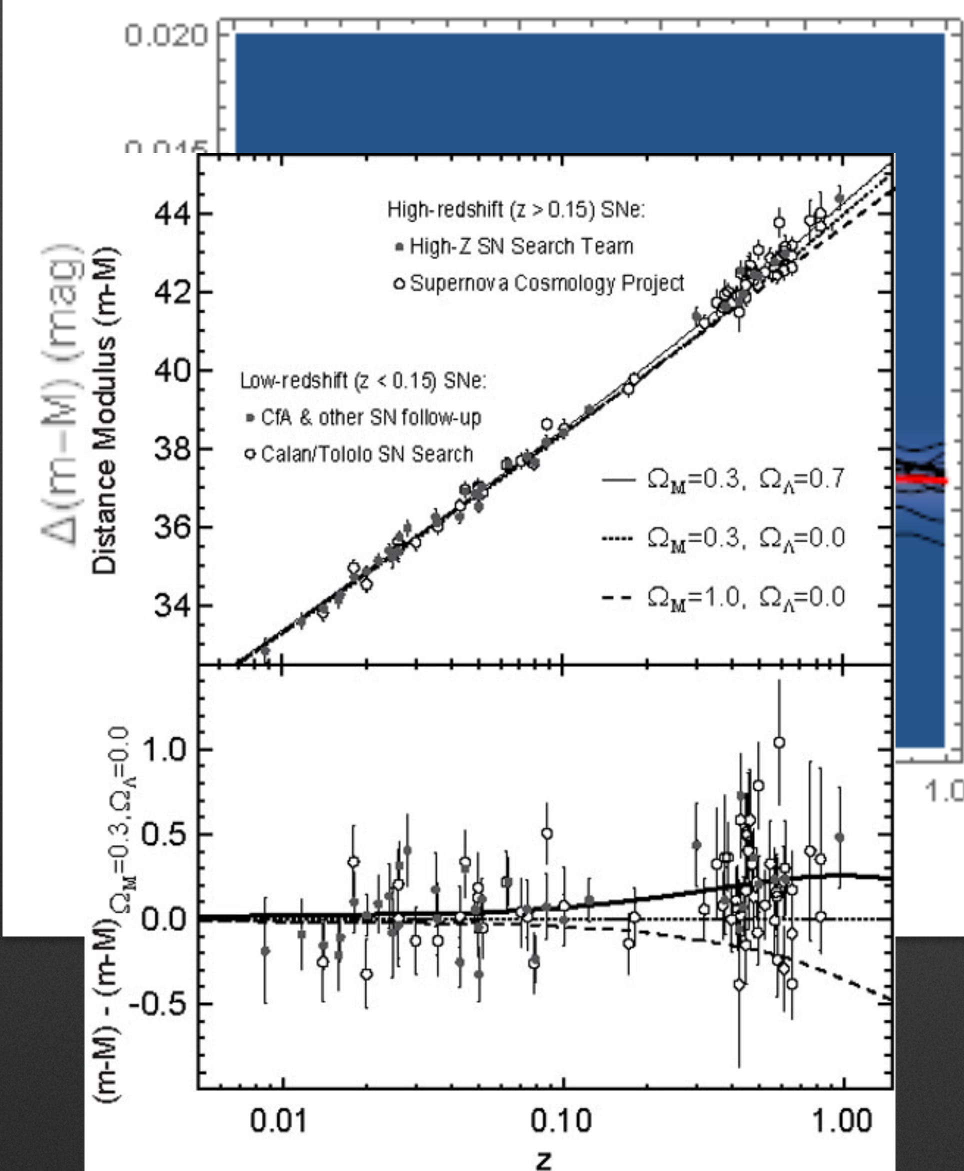
We can go out further

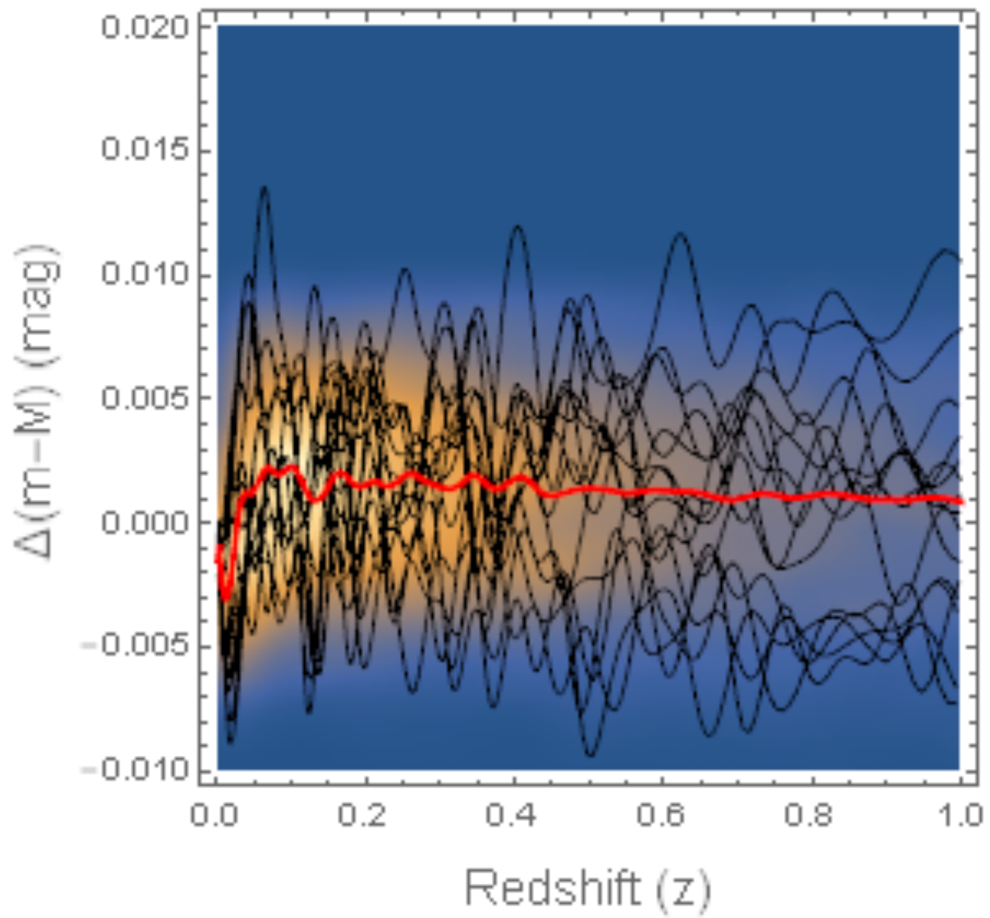


$$\frac{\sigma_\rho}{\rho} = 0.03$$

$$\frac{\sigma_\rho}{\rho} = 0.19$$

We can go out further





$$\frac{\sigma_\rho}{\rho} = 0.19$$

How do we parameterize success?

- Reproducing GR requires the additional satisfaction of a set of constraints
  - The Hamiltonian Constraint:

$$\mathcal{H} \equiv \bar{\gamma}^{ij}\bar{D}_i\bar{D}_j e^{\phi} - \frac{e^{\phi}}{8}\bar{R} + \frac{e^{5\phi}}{8}\tilde{A}_{ij}\tilde{A}^{ij} - \frac{e^{5\phi}}{12}K^2 + 2\pi e^{5\phi}\rho = 0$$

  - The Momentum Constraints:

$$\mathcal{M}^i = \bar{D}_j(e^{6\phi}\tilde{A}^{ij}) - \frac{2}{3}e^{6\phi}\bar{D}^i K - 8\pi e^{10\phi}S^i = 0$$

- While the BSSN method is analytically equivalent to GR, the numerical implementation can still propagate spurious solutions if you leave the constraint surface

Are we doing things right?

History says that we need to test the robustness of the code before we can extract any true results from it

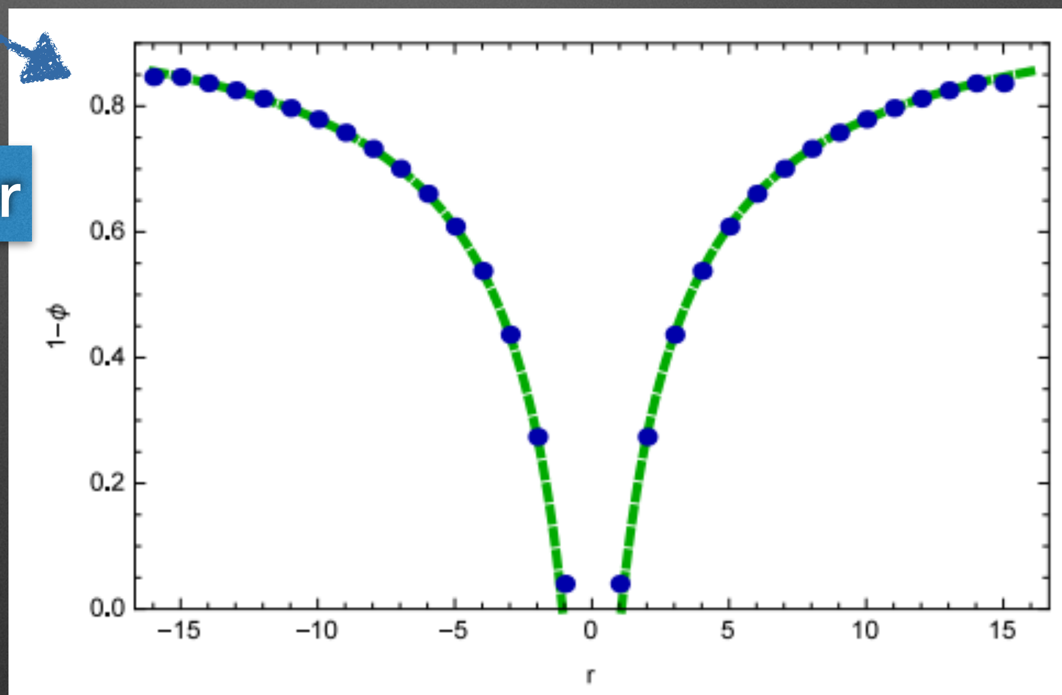
“Apples to/with Apples”

- To show that we trust our numerical implementation, we run a set of standard tests and parameterize how “well” we do:
    - Can we recreate a black hole?
    - Can we recreate gravitational waves?
    - Can we recreate the homogeneous Universe?

Code Tests: Black Hole*

$$\mathcal{H} \equiv \bar{\gamma}^{ij}\bar{D}_i\bar{D}_j e^{\phi} - \frac{e^{\phi}}{8}\bar{R} + \frac{e^{5\phi}}{8}\tilde{A}_{ij}\tilde{A}^{ij} - \frac{e^{5\phi}}{12}K^2 + 2\pi e^{5\phi}\rho$$

Black Hole Behavior

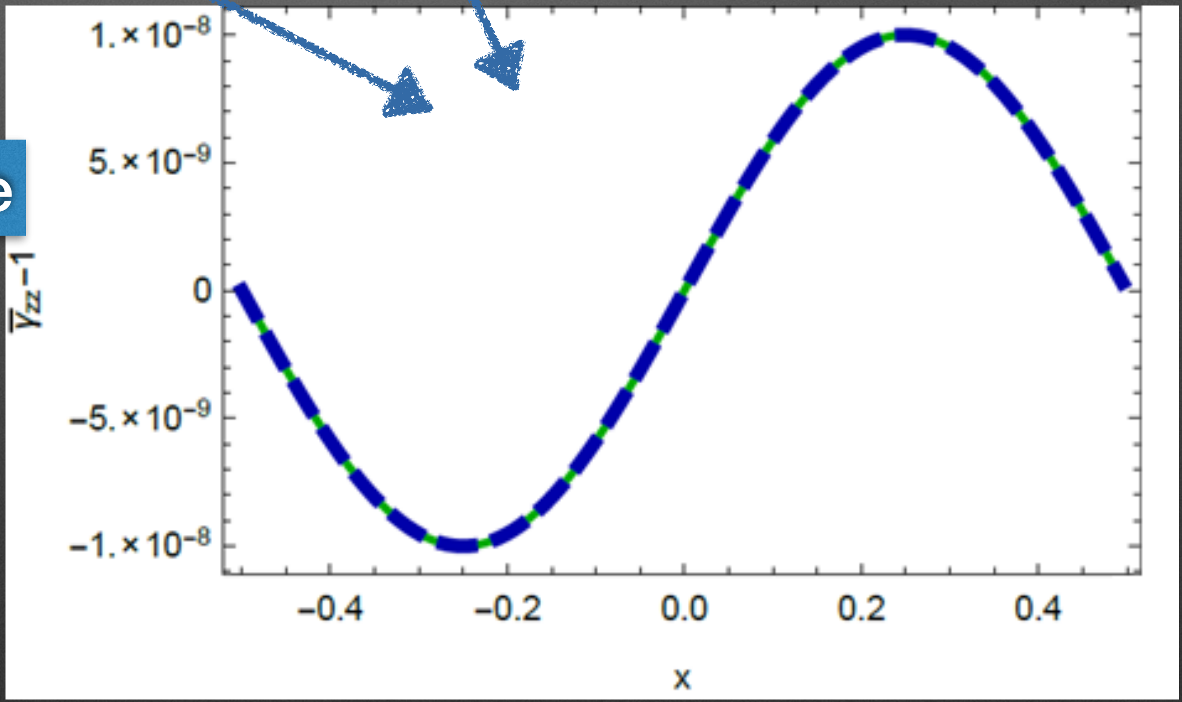


*we use “1+Log” gauge/slicing for this simulation

Code Tests: Gravitational Wave

$$\mathcal{H} \equiv \bar{\gamma}^{ij}\bar{D}_i\bar{D}_j e^{\phi} - \frac{e^{\phi}}{8}\bar{R} + \frac{e^{5\phi}}{8}\tilde{A}_{ij}\tilde{A}^{ij} - \frac{e^{5\phi}}{12}K^2 + 2\pi e^{5\phi}\rho$$

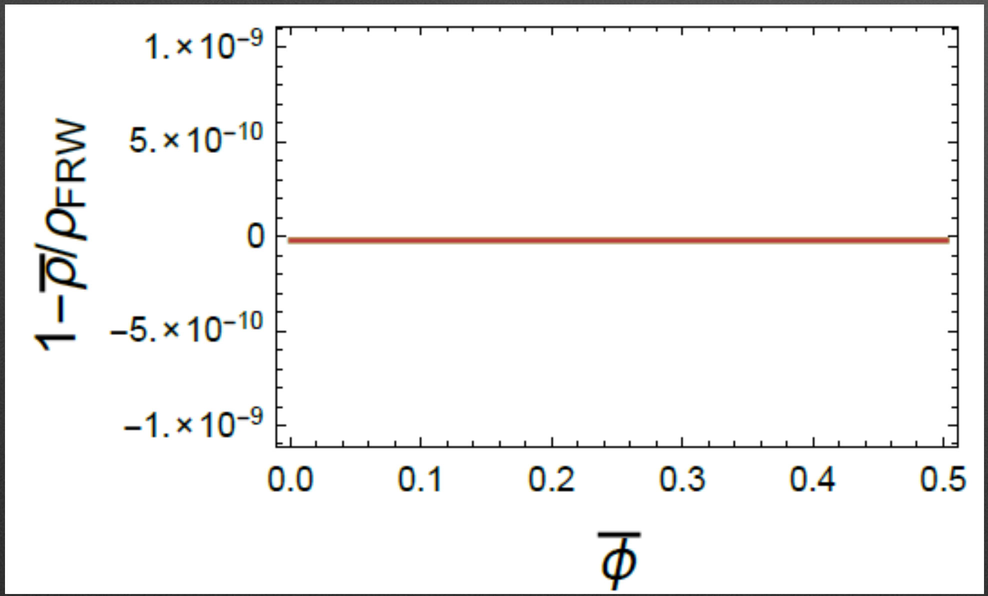
Linearized wave

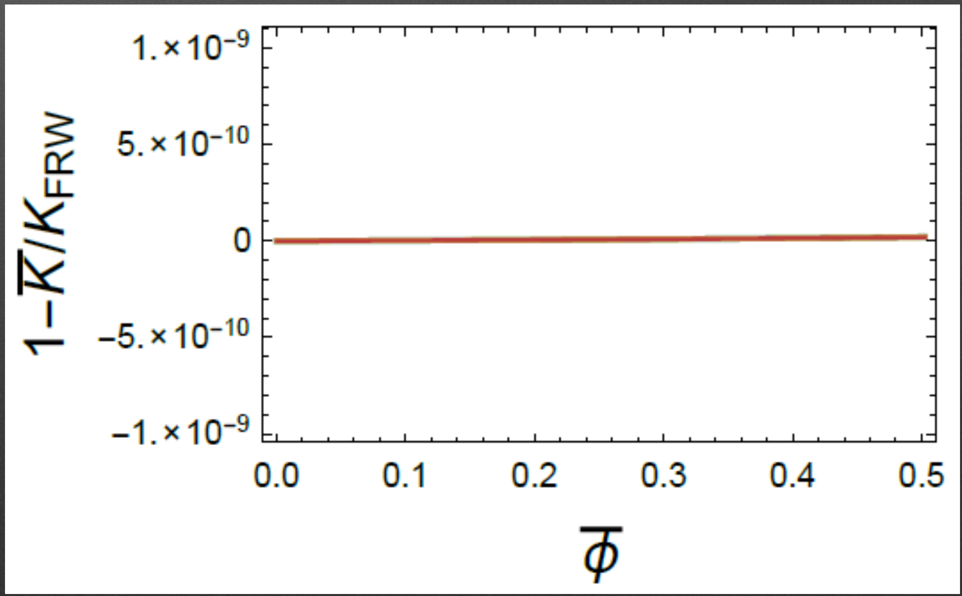


Code Tests: FLRW

$$\mathcal{H} \equiv \bar{\gamma}^{ij}\bar{D}_i\bar{D}_j e^{\phi} - \frac{e^{\phi}}{8}\bar{R} + \frac{e^{5\phi}}{8}\tilde{A}_{ij}\tilde{A}^{ij} - \frac{e^{5\phi}}{12}K^2 + 2\pi e^{5\phi}\rho$$

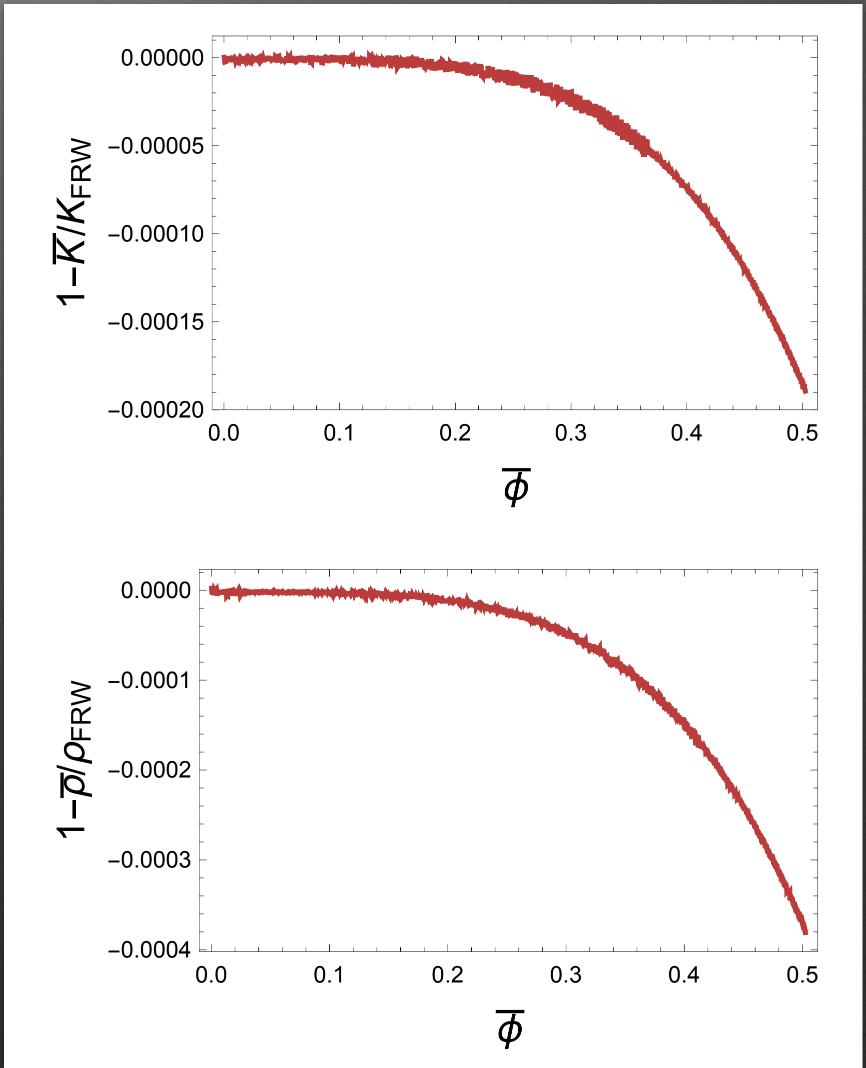
The homogeneous Universe





Background Expansion

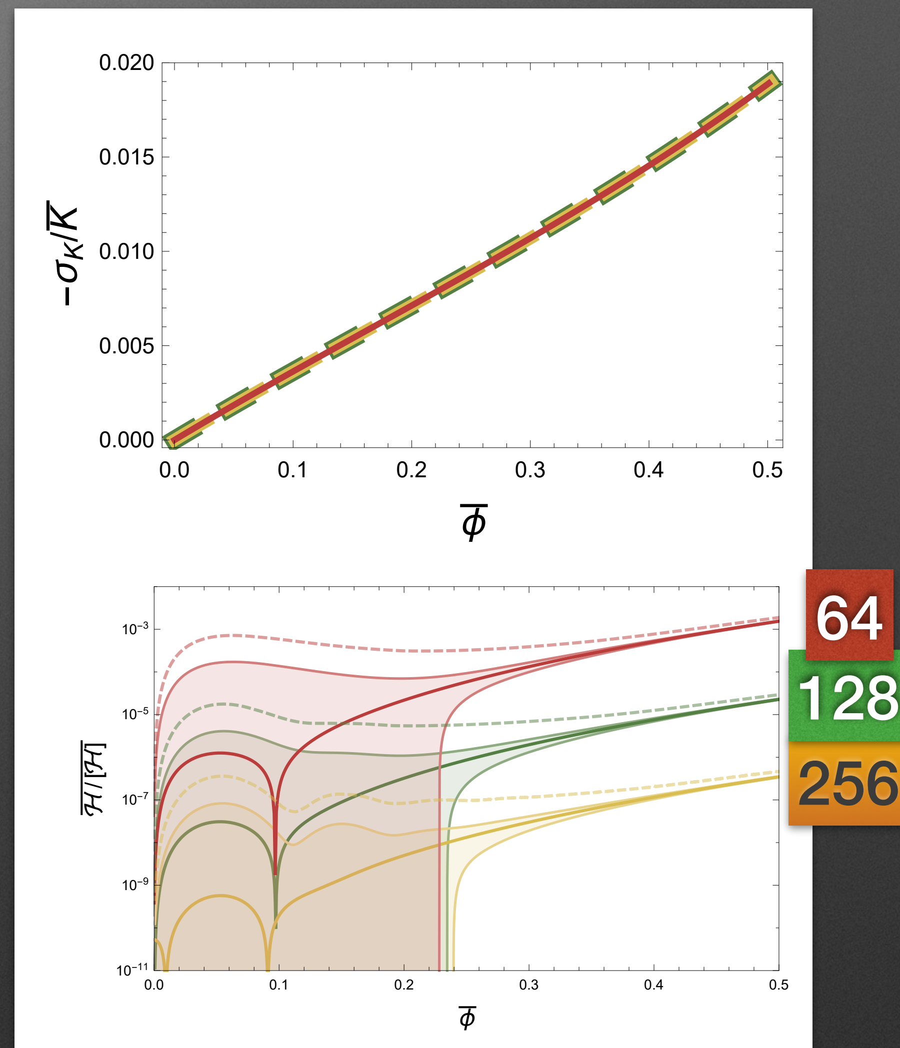
- The *average* values of the density and extrinsic curvature follow the *exact* FLRW expectation



Question #2
Is the Universe inhomogeneous at small scales?

We'll be interested in the generation of inhomogeneity of *K*

- For the Fiducial model, this (converges) and is resolution independent (given the same *physical* box size, power spectrum peak and cutoff)

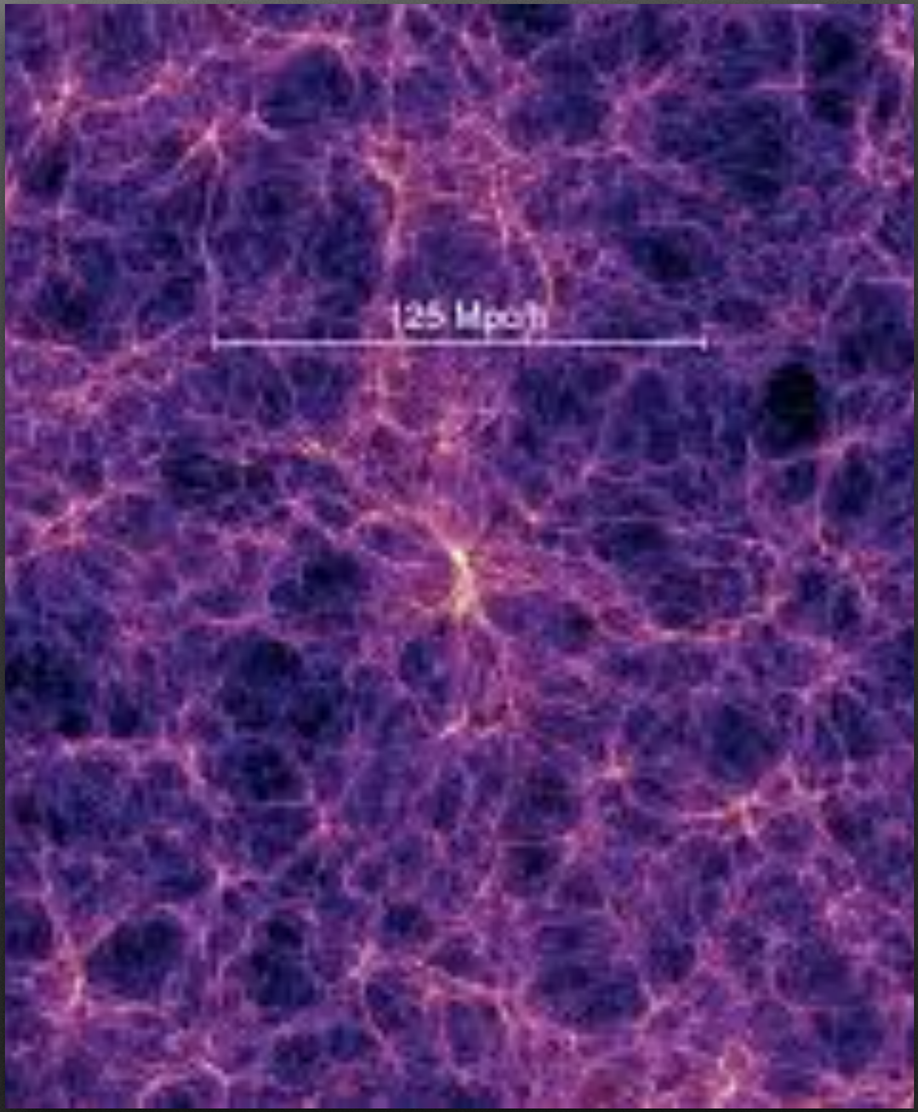


Weren't you going to talk about physics?

- So we have a numerical framework
- Let's start a simulation where we have a volume of the Universe with some density perturbations

$$P_k = \frac{4P_*}{3}\frac{k/k_*}{1+(k/k_*)^{4/3}}$$

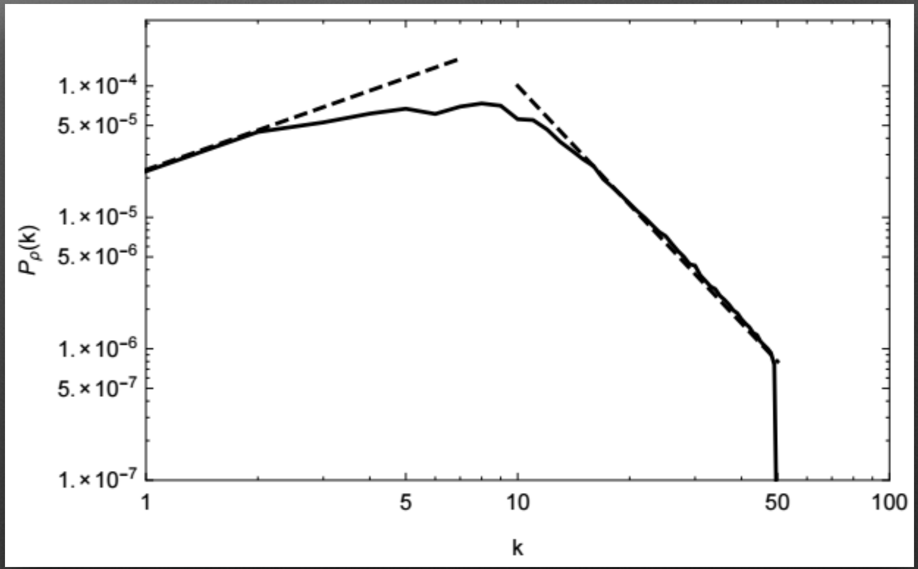
- Then solve the initial condition problem (and put all the inhomogeneities in the volume elements *not* the expansion rates)



The initial value problem

- By whatever means necessary, we begin with the assumption of homogeneous extrinsic curvature, and the metric response (to the source) is just in the conformal factor,

$$\psi \equiv e^{\phi}$$



- So that the initial conformal factor must obey the following situation,

$$\rho = \rho_K + \rho_\psi$$

$$\nabla^2\psi = -2\pi\psi^5\rho_\psi$$

$$K = -\sqrt{24\pi\rho_K}$$

Fiducial Model

$$N^2 = 128^3$$

$$\Delta t = \frac{\Delta x}{10}$$

$$L = \frac{H_I^{-1}}{2}$$

$$k_{\rm peak} = \frac{7}{128}\Delta k$$

$$k_{\rm cutoff} = \frac{10}{128}\Delta k$$

$$\frac{\sigma_\rho}{\rho} = 0.04$$