Primordial trispectrum and modulations in the CMB

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Inflation $\rightarrow$ Primordial fluctuations, $\Phi(x)$

$\Phi(x) \rightarrow$ angular power spectrum, $C_\ell$s

Motivation
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Inflation $\rightarrow$ Primordial fluctuations, $\Phi(\mathbf{x})$

$\Phi(\mathbf{x}) \rightarrow$ angular power spectrum, $C_\ell$

Generally assumed that primordial fluctuations are Gaussian.

Planck has put strict constraints on certain shapes of primordial non-Gaussianity. For e.g. the local-type bispectrum: $f_{\text{NL}}^{\text{local}}$. 
Motivation

- Non-Gaussianity that couples different scales are not possible in simplest models of inflation.

\[
\langle \Phi(k_1)\Phi(k_2)\Phi(k_3) \rangle \quad \langle \Phi(k_1)\Phi(k_2)\Phi(k_3)\Phi(k_4) \rangle
\]
Non-Gaussianity that couples different scales are not possible in simplest models of inflation.

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Level of non-Gaussianity: $f_{NL}\sqrt{A_\Phi}$ and $\tau_{NL}A_\Phi$. 
Planck 2013 $\tau_{NL}$ result: $\tau_{NL} < 2800$ was obtained using not a four-point function estimator but using modulations:

$$\Phi(x) = \zeta(x)(1 + \phi(x))$$

induces a four-point function that peaks in the collapsed configuration.

$$\propto \tau_{NL} P_{\Phi}(k_1) P_{\Phi}(k_3) P_{\Phi}(|k_1 - k_2|)$$

Planck team reconstructed the modulating field and used its power spectrum to constrain the amplitude $\tau_{NL}$. 
The effect of such a modulation in the CMB temperature field is

\[ \frac{\Delta T}{T}(\hat{n}) = \left[ \frac{\Delta T}{T}(\hat{n}) \right]_{\text{iso}} \left( 1 + \sum_{LM} \phi_{LM} Y_{LM}(\hat{n}) \right) \]

The modulation is largest at low \( L \) for a scale invariant modulating field.
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The modulation is largest at low \( L \) for a scale invariant modulating field. Also, the modulation is constant, means constraints are dominated by large \( \ell \) (fluctuations at smaller scales) data and not sensitive to scale-dependent signal.
A different collapsed-limit trispectrum model

The collapsed limit of the trispectrum of the quasi-single field model – 0911.3380 (Chen and Wang) 1204.4207 (Assassi et. al)

\[ \propto \tau_{NL} P_\Phi(k_1) P_\Phi(k_3) P_\Phi(|k_1 - k_2|) \left( \frac{|k_1 - k_2|}{\sqrt{k_1 k_3}} \right)^{3-2\nu} \]
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Position-dependent power spectrum

\[ P(k, \mathbf{x}) \approx P(k) \left[ 1 + 4\sqrt{\tau_{NL}} \int \frac{d^3 q}{(2\pi)^3} \Phi(q) \left( \frac{q}{\sqrt{k}} \right)^{3/2-\nu} e^{i q \cdot \mathbf{x}} \right] \]

which when projected on the CMB sky, implies scale-dependent modulation of the power spectrum:

\[ P(k, \hat{n}) \approx P(k) \left( 1 + \sum_{LM} g_{LM}(k) Y_{LM}(\hat{n}) \right) \]

Only the variance \[ \langle g_{LM}(k)^2 \rangle \] can be computed.
Position-dependent power spectrum

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which when projected on the CMB sky, implies scale-dependent modulation of the power spectrum:

\[ P(k, \mathbf{\hat{n}}) \approx P(k) \left( 1 + \sum_{LM} g_{LM}(k) Y_{LM}(\mathbf{\hat{n}}) \right) \]

Only the variance \( \langle g_{LM}^2(k) \rangle \) can be computed.

\[ \langle g_{LM}^2(k) \rangle = 64\pi \tau_{\text{NL}} \int \frac{dq}{q} j_L^2(qr) P_\Phi(q) \left( \frac{q}{\sqrt{k}} \right)^{3-2\nu} \]
Expected scale-dependent modulation amplitudes

\( \tau_{NL} = 10^5, \nu = 1.2 \)

\[ \langle g_{LM}^2 \rangle^{0.5} \]

- Blue line: \( L = 1 \)
- Orange dotted line: \( L = 2 \)

For \( \tau_{NL} = 2800, \nu = 1.5 \), \( L = 1 \).
Example: Dipole modulation

\[ \hat{A}(\ell) = \frac{1}{(2\ell + 1) \sqrt{C_\ell C_{\ell+1}}} \sum_{m=-\ell}^{\ell} a^*_{\ell m} a_{\ell+1,m} \]
Example: Dipole modulation

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Then, the covariance depends on the trispectrum:

\[ \langle \hat{A}^*(\ell) \hat{A}(\ell') \rangle = \frac{1}{C_\ell C_{\ell+1}} \langle a_{\ell m} a_{\ell+1, -m} a_{\ell' -m', -m'} a_{\ell' +1, m} \rangle \]

computation of the T/E trispectrum is currently being worked on.
Hints of scale-dependent modulations in the CMB temperature fluctuations can be described in terms of a primordial trispectrum.

The modulation measurements can in turn be used to constrain the amplitude of collapsed-limit of primordial four-point functions.

- **detection**: consequences for inflationary model building.
- **tighter constraints**: increases our confidence in the assumption that primordial fluctuations are Gaussian.

**work in progress** – Full CMB temperature and polarization trispectrum at different modulation multipoles $L$ which will allow us to forecast constraints on $\tau_{NL}, \nu$. 
\[ H = \frac{1}{2} (\partial_\mu \theta)^2 + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} m^2 \sigma^2 - \rho \dot{\theta} \sigma + \mu \sigma^3 \quad (1) \]

- At linear order, \( \theta = -\sqrt{2\epsilon} M_{\text{pl}} \zeta \) (1204.4207)
- \( \sigma \) is the extra field whose mass \( m \) determines the squeezed and collapsed limit scaling of the bispectrum and trispectrum.