Primordial trispectrum and modulations in the CMB

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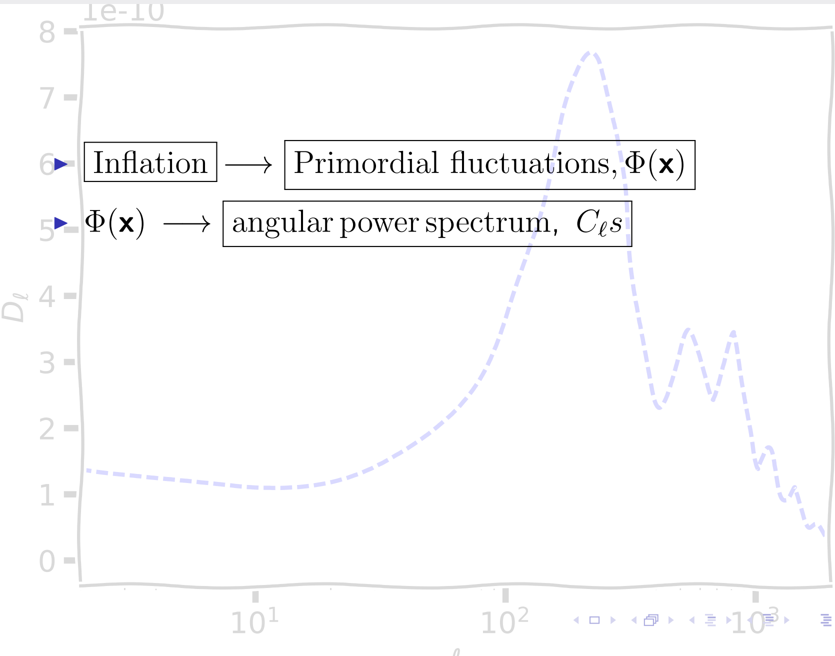
Motivation

- Inflation $\longrightarrow$ Primordial fluctuations, $\Phi(\mathbf{x})$
- $\Phi(\mathbf{x}) \longrightarrow$ angular power spectrum, $C_\ell s$



Motivation

- Inflation $\longrightarrow$ Primordial fluctuations, $\Phi(\mathbf{x})$
- $\Phi(\mathbf{x}) \longrightarrow$ angular power spectrum, $C_\ell s$
- Generally assumed that primordial fluctuations are Gaussian.
- Planck has put strict constraints on certain shapes of primordial non-Gaussianity. For e.g. the local-type bispectrum: $f_{\rm NL}^{\rm local}$.

Motivation

- Non-Gaussianity that couples different scales are not possible in simplest models of inflation.

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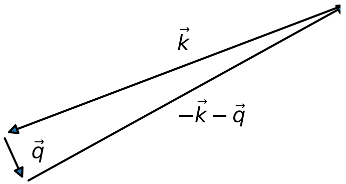

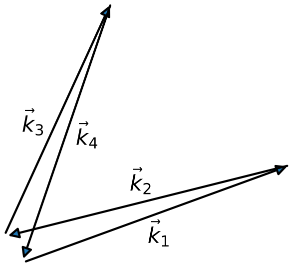

Level of non-Gaussianity: $f_{\rm NL}\sqrt{A_\Phi}$ and $\tau_{\rm NL}A_\Phi$.

Modulation estimator for trispectrum

- Planck 2013 $\tau_{\rm NL}$ result: $\tau_{\rm NL} < 2800$ was obtained using not a four-point function estimator but using modulations:

$$\Phi(\mathbf{x}) = \zeta(\mathbf{x})\,(1 + \phi(\mathbf{x}))$$

induces a four-point function that peaks in the collapsed configuration.

$$\boxed{\propto \tau_{\rm NL} P_\Phi(k_1) P_\Phi(k_3) P_\Phi(|\mathbf{k}_1 - \mathbf{k}_2|)}$$

- Planck team reconstructed the modulating field and used its power spectrum to constrain the amplitude $\tau_{\rm NL}$.

Effect on the CMB fluctuations

- The effect of such a modulation in the CMB temperature field is

$$\frac{\Delta T}{T}(\hat{n}) = \left[\frac{\Delta T}{T}(\hat{n})\right]_{\rm iso}\left(1 + \sum_{LM} \phi_{LM} Y_{LM}(\hat{n})\right)$$

The modulation is largest at low L for a scale invariant modulating field.

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The modulation is largest at low L for a scale invariant modulating field. Also, the modulation is constant, means constraints are dominated by large ℓ (fluctuations at smaller scales) data and not sensitive to scale-dependent signal

A different collapsed-limit trispectrum model

The collapsed limit of the trispectrum of the quasi-single field model – 0911.3380 (Chen and Wang) 1204.4207 (Assassi et. al)

$$\propto \tau_{\rm NL} P_\Phi(k_1) P_\Phi(k_3) P_\Phi(|\mathbf{k}_1 - \mathbf{k}_2|) \left(\frac{|\mathbf{k}_1 - \mathbf{k}_2|}{\sqrt{k_1 k_3}} \right)^{3-2\nu}$$

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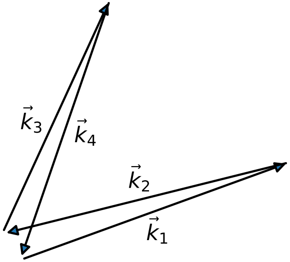

Position-dependent power spectrum

$$P(k,\mathbf{x}) \approx P(k)\left[1 + 4\sqrt{\tau_{\rm NL}}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\Phi(\mathbf{q})\left(\frac{q}{\sqrt{k}}\right)^{3/2-\nu} e^{i\mathbf{q}\cdot\mathbf{x}}\right]$$

which when projected on the CMB sky, implies scale-dependent modulation of the power spectrum:

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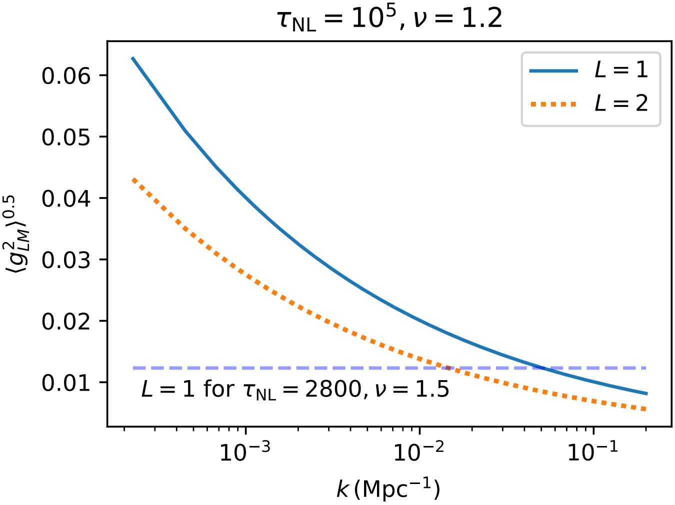
Only the variance $\langle g_{LM}^2(k) \rangle$ can be computed.

$$\langle g_{LM}^2(k) \rangle = 64\pi\tau_{\rm NL} \int \frac{dq}{q} j_L^2(qr) \mathcal{P}_\Phi(q) \left(\frac{q}{\sqrt{k}} \right)^{3-2\nu}$$

Expected scale-dependent modulation amplitudes



Working towards a full CMB analysis

Example: Dipole modulation

$$\hat{A}(\ell) = \frac{1}{(2\ell+1)\sqrt{C_\ell C_{\ell+1}}} \sum_{m=-\ell}^{\ell} a^*_{\ell m} a_{\ell+1,m}$$

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Example: Dipole modulation

$$\hat{A}(\ell) = \frac{1}{(2\ell+1)\sqrt{C_\ell C_{\ell+1}}} \sum_{m=-\ell}^{\ell} a^*_{\ell m} a_{\ell+1,m}$$

Then, the covariance depends on the trispectrum:

$$\langle \hat{A}^*(\ell)\hat{A}(\ell')\rangle = \frac{1}{C_\ell C_{\ell+1}} \langle a_{\ell m} a_{\ell+1,-m} a_{\ell',-m'} a_{\ell'+1,m}\rangle$$

computation of the T/E trispectrum is currently being worked on.

Summary

- Hints of scale-dependent modulations in the CMB temperature fluctuations can be described in terms of a primordial trispectrum
- The modulation measurements can in turn be used to constrain the amplitude of collapsed-limit of primordial four-point functions.
  - detection: consequences for inflationary model building.
  - tighter constraints: increases our confidence in the assumption that primordial fluctuations are Gaussian
- work in progress – Full CMB temperature and polarization trispectrum at different modulation multipoles L which will allow us to forecast constraints on $\tau_{\rm NL}, \nu$.

Extra slide: QSFI Hamiltonian

$$H = \frac{1}{2}(\partial_\mu\theta)^2 + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}m^2\sigma^2 - \rho\dot{\theta}\sigma + \mu\sigma^3 \qquad (1)$$

- At linear order, $\theta = -\sqrt{2\epsilon}M_{\rm pl}\zeta$ (1204.4207)
- σ is the extra field whose mass m determines the squeezed and collapsed limit scaling of the bispectrum and trispectrum.